The Process

My Intel Science Talent Search project was a four year labor of love that tested my patience more than any undertaking that I had ever attempted in my life. Once, after a particularly frustrating day in the lab, I told my mentor how upset I was about spending the previous three hours wiring a cryogenic thermometer only to have it break as it was inserted into the liquid helium Dewar. "Sounds like you're doing research," he replied. This incident was emblematic of my entire research project. For almost two years, I researched low temperature superconductors before realizing that I would not be able to conduct a valid experiment, and I was forced to change my topic in the middle of my junior year. I spent a hectic final nine months collecting data and writing the paper. Looking back, I realize that my project taught me two very important lessons: one, the power of perseverance and determination and; two, how *not* to conduct a research project.

The unfortunate situation that I found myself in during my junior year was not so much the result of poor planning than the simple fact that I had underestimated my task. When I was first instructed as a freshman to choose a research topic, I was advised to pick a subject that could understood and experimented on by a high school student. But more importantly, as I was told by my research adviser, "nothing is impossible if you set your mind to it." Perhaps, as an already occupied high school student learning the physics and math that describe superconductivity in four years is not impossible, but it is safe to say that it presents some problems. At best, my foray into superconductors was ill-advised, but it nevertheless led to the most fortuitous aspect of my research project: my mentor Dr. John R. Kirtley.

I came across Dr. Kirtley's name when looking through a list of ongoing projects at

IBM's Thomas J. Watson Research Center in Yorktown Heights, New York. This massive lab was only 15 minutes from my home and provided an ideal setting to conduct a research project. Although I did not know it at the time, Dr. Kirtley was an accomplished researcher, having been a research staff member at IBM for over 25 years, an elected fellow of the American Physical Society, and having been awarded the prestigious Oliver E. Buckley Condensed Matter Physics Prize. Dr. Kirtley had been researching superconductors for several decades and specialized in something called superconducting quantum interference device (SQUID) microscopy. At first, Dr. Kirtley was reluctant to take me on as a student. He knew that I would not be able to understand how a SQUID worked, let alone use it. I assured him that I would be able to design and carry out my own experiment and although he was somewhat skeptical, he agreed to be my mentor.

The first task Dr. Kirtley assigned to me was to determine the transition temperature (the temperature at which an object changes to a superconducting state) of a certain object. He showed me how to use the equipment I would need and how to record the data, but otherwise, he left me alone to carry out the experiment. I was ecstatic about the freedom I had in conducting research because when Dr. Kirtley was not in the lab I had the opportunity to design my own experiments; namely, "Will this break if I dunk it nitrogen and drop it on the floor?" I had a terrific time doing my early research on superconductivity except for the fact that I needed a new pair of sneakers.

During my sophomore year I began to develop ideas for an experiment for my science research project. Some of them were far-fetched, like using superconductors to shield gravity. Some of them were trivial, like the effect of superconductivity on capacitance (there is none). The difficulties I encountered in developing a feasible yet meaningful project were due to the fact that the so-called conventional superconductors had been thoroughly researched since their discovery a century ago, and the current research that was going on in the field was inaccessible to me because of my lack of education. I suddenly found myself midway through my junior year with nothing to show for my two years of research.

After several weeks of brainstorming, Dr. Kirtley and I came to the conclusion that I would have to change topics entirely or drop out of the program. I had no desire to come out empty handed after all the work I had done, but I had no idea what I would research now that superconductors were out of the picture. It was Dr. Kirtley that suggested that I take a look at some research that was going on in the field of packing. He showed me an article about some physicists at Princeton who conducted a study on the efficiency of packing M & Ms. He then showed me a paper about a mathematician from the University of Michigan who had just proved a 400 year old problem called the Kepler Conjecture, which asked for the most efficient way to pack spheres. After seeing this I laughed. How could I, having worked with materials just four degrees above absolute zero, using lock-in amplifiers and capacitance bridges and cryogenic thermometers, be expected to do a research paper on such a mundane topic? But time was running out and I begrudgingly accepted that this was my only option.

Even though I had picked a new research topic, I still did not know what my specific experiment would be. What exactly would my research be about? Should I try to pack different shapes or use different containers? Would my research have any practical applications? A breakthrough occurred when I was telling a friend about the problem and he likened it to the way pellets are packed in a shotgun shell. I looked into this idea and it turned out that there were industrial research papers dating as far back as 1929 that dealt with the topic. I also found out that there was a new project being funded by DARPA to develop electronic weapons which packed the rounds directly on top of each other in cylindrical canisters. Although I did not intend to create a new generation of weapons, this was enough to attract my interest and from this I produced my research question: What is the best way to pack spheres in a cylinder?

At first, work was slow and I was faced with several problems. I did not know how to approach the issue from a research standpoint; I could use physical experimentation to measure the efficiency of placing spheres in a cylinder, or I could attempt to create a packing scheme using algebraic and geometric calculations. Again Dr. Kirtley came to the rescue by answering the question easily. “Do both,” he said.

My research project was coming together. Instead of messing around with fancy equipment or performing experiments that I did not understand, I was now developing my own interpretation of a problem using only my brain and a calculator. It was incredibly gratifying when I began to make legitimate discoveries. I will never forget when I first created a function relating the height and number of spheres within a cylinder to the efficiency of the packing and realized that, at that moment, I was only person on Earth who understood this relationship. I began to design and build a shake-table which would be my main experimental apparatus. In the middle of the summer before my senior year I taught myself the Visual Basic programming language in three days so that I could develop a simple application that would create data for graphical purposes. I also began taking calculus at Pace University over the summer so that I could understand the math behind my project, which often required the concept of limits.

As far as packing problems are concerned, there is no such thing as an efficiency greater than 100 percent, meaning that all the space in a given area or volume is occupied. Early in my research I realized that the efficiency function I created yielded a finite value as the argument approached infinity. This was to be expected, considering the nature of the packing, but often I was unable to calculate my answers because of my inexperience in math. The calculus course gave me the tools I needed to solve the problem. I utilized L'Hôpital's rule frequently in my paper to deal with limits, especially in the main result of my paper.

I learned more about research in the summer before my senior year than in my previous three years working at Watson Research Center. By the time I started writing my paper in late August, I felt confident that I had done all I could to salvage the project. More importantly, I felt, was that I had learned the joys and pitfalls of science research and gained valuable insight into a possible career. I was not without my regrets, but I was proud of the work I had done and proud that I had even finished such a difficult task. Anything else was just icing on the cake.

The Paper

My project combined areas of physics, mathematics, and engineering in order to solve an ostensibly simple, yet practically difficult problem. Ideally, one would use pure mathematics to determine the best way to pack spheres in a cylinder. However, pure mathematics requires that you *prove* that your method results in the most efficient packing. It took mathematicians 400 years to prove the much simpler Kepler Conjecture, so for my project pure mathematics was out of the question. Instead, I opted to use as much intuitive reasoning as possible, using mathematics to back the ideas I came up with, while supplementing those ideas with physical experimentation and data produced from computer applications. The project was unique in that it used a variety of approaches to handle the same problem. The results from each result could then be compared to determine the superior packing.

All of the packings that were derived from the mathematical approach in the paper are based on the two-dimensional arrangement of equal disks in a circle. These arrangements become the three-dimensional “bases” for packing spheres in cylinders. The basis of the research is that layering the best known two-dimensional arrangements of equal disks in a circle should result in the best known three-dimensional packings of spheres in cylinders. Optimally, one would like to produce a packing that approaches the density of the Kepler Conjecture, roughly 74 percent.

To optimize the packing of spheres in a cylinder, initial considerations included what the proper dimensions of the container should be for the size and number of spheres to be packed. There are, of course, an infinite number of cylinders that can be created but not all of them deserve consideration. The ones that can be used to form dense packings are those which are worth investigating, and in all likelihood these are ones in which the base layer fits spheres as shown in the examples below. This is the equivalent of packing two-dimensional disks in a circumcircle, and this is how the exact dimensions of the bases of the cylinders used for this project were chosen. The two-dimensional packings were based on the results of many years of geometrical investigations by mathematicians. The packings of one, two, three, and four equal circles (Fig. 1) within a larger circle are each considered to be self-evident that they are indeed the optimal packings (Kravitz). Optimal packings from 5 to 11 disks in a circle have each been proved. No packing of 12 or more circles have been proved to be the most efficient. In recent years computer algorithms have been used to find tight two-dimensional packings of up to 65 disks in a circle (Graham et al.).

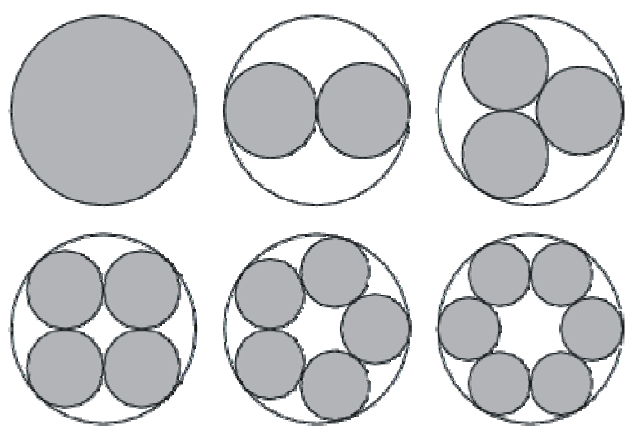


Figure 1

The first five two-dimensional arrangements of disks in a circle, as shown in figure 1, are very straightforward and did not require computer simulation or experimental testing to show the manner in which the bases should be layered. The efficiency of the packings of bases 1-5 are all easily described by the ratio φ of the volume of the spheres with radius 1 (r = 1) V_s, divided by the volume of the cylinder, V_c :

$$\varphi = \frac{V_s}{V_c} = \frac{NL\left(\frac{4}{3}\pi r^2\right)}{\pi R^2\left(H(L-1)+2\right)} = \frac{4NL}{3R^2\left(H(L-1)+2\right)}$$

where N is the number of spheres per level, L is the number of levels, R is the radius of the encompassing circle, and H is the additional height per level. This equation is a function of L. The values of N, R, and H all depend on the arrangement of the base layer spheres. This section of the research was the only area in which no physical experimentation occurred. Again, intuitive reasoning led me to believe that this was indeed the most efficient way to pack spheres in a cylinder, although I would be hard pressed to prove it.

The next aspect of my research involved looking at a unique set of base layer arrangements called the curved hexagonal series, $h(k) = 3k(k + 1) + 1$ (Fig. 2). In these two-dimesional packings, the circles align themselves with one of six curved arms extending from the center (Lubachevsky and Graham). These packings mimic the most efficient packing of circles in a plane which is in the form of a basic honeycomb

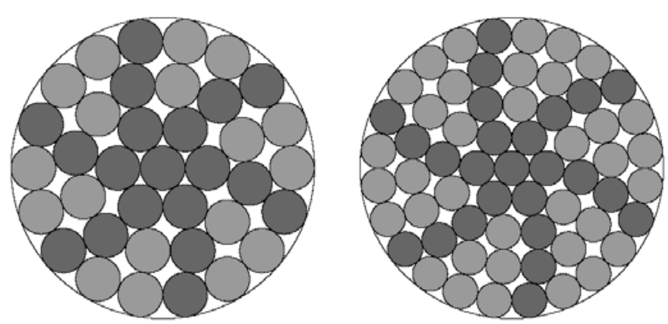
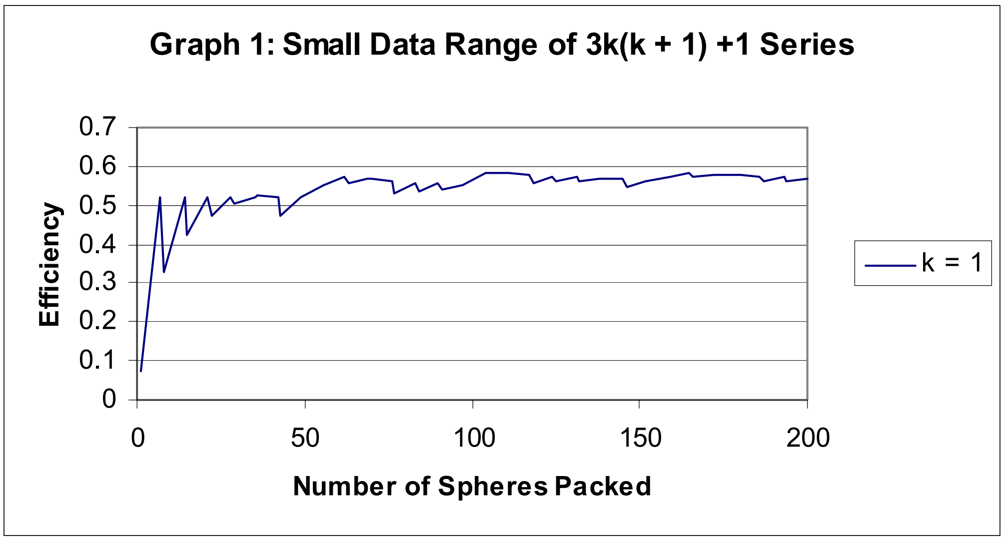
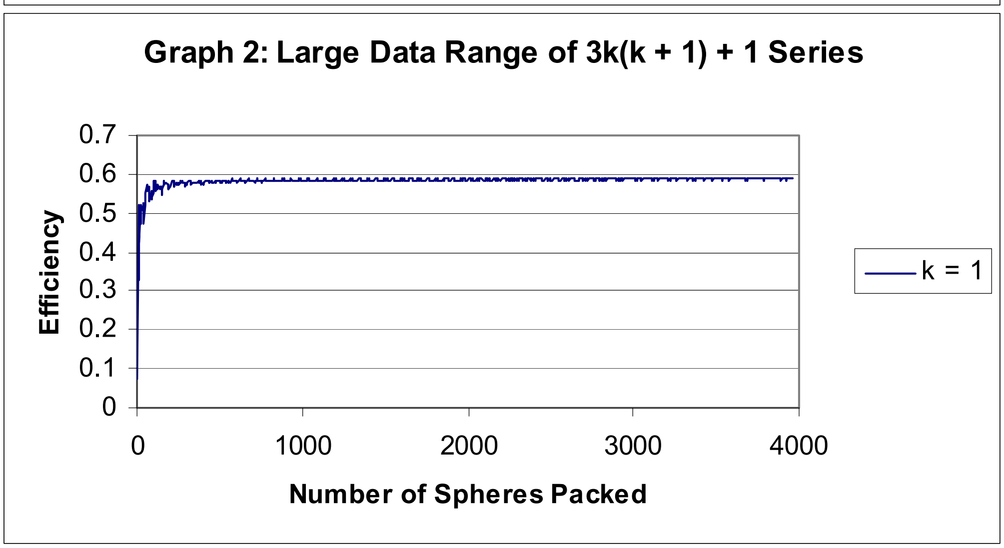


Figure 2

pattern. Because these packings seem to be the most efficient in two-dimensions, it was my hypothesis that they could be used to create the most efficient packing of spheres in cylinders.

To demonstrate this, I used two methods. The first was a computer application which functioned by layering the inner and outer layers separately because they do not layer evenly. I used the data created by this program to create the following graphs.





The most noticeable aspect about the graph was that despite the initial irregularity, it was asymptotic, meaning that it levels out to a finite value. Unfortunately, my computer program could not precisely compute that value, so again I used mathematics to find the answer.

The main result of my paper was an equation that computed the efficiency of a curved hexagonally packed cylinder with infinite height and infinite width as described by the following equation:

$$\varphi(k)=\frac{4(3k(k+1)r+s)}{6\left(1+\csc\left(\frac{\pi}{6k}\right)\right)^2 s}=\frac{4\left(k^2+k\right)\sin^2\left(\frac{\pi}{6k}\right)}{\sqrt{3}\left(\sin\left(\frac{\pi}{6k}\right)+1\right)^2}+\frac{2}{3\left(\csc\left(\frac{\pi}{6k}\right)+1\right)^2}$$

After several manipulations and applying L'Hôpital's rule I was able to come up with the finite solution to the problem $\frac{\pi^2\sqrt{3}}{27}\approx 63\%$. This answer made sense because it was less than the

Kepler Efficiency but greater than that of any other packing investigated. It was therefore my conclusion that the highest known efficiency for packing spheres in a cylinder was 63 percent.

The final aspect of my project was purely experimental. I investigated the packing of base layer 8 (Fig. 3) by first measuring the efficiency of placing ball-bearings in a cylinder by hand and then comparing it to a random packing using a shake table that I built. I ran several trials for each additional ball-bearing packed and I recorded the height of the balls in the cylinder to determine an efficiency. The results confirmed my hypothesis that the hand-layered packing was generally more efficient that the random packing, as shown in graph 3. My project was generally successful because it went beyond the hypothesis to propose solutions to the problem and did so in a way that had applications in both

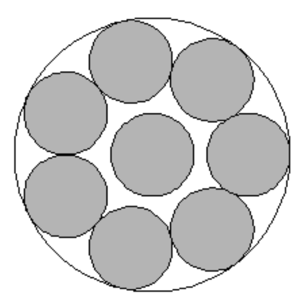
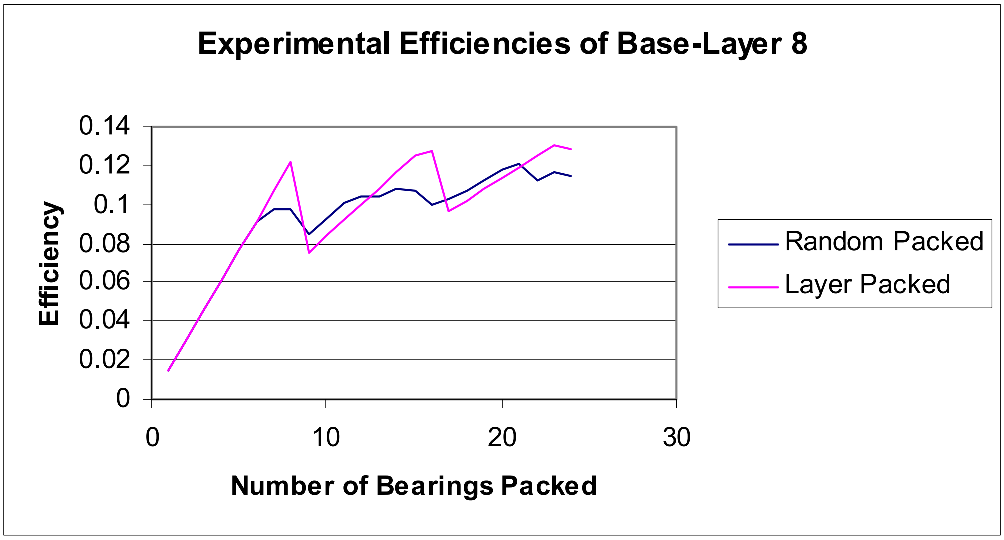


Figure 3

mathematical and physical study.



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