Determining a value for gravity with an accuracy of 10 parts per billion for the Electronic Kilogram Experiment

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I. Personal

I have a background in biological research, specifically in the fields of molecular and behavioral genetics. In fact, my research project for the Intel Science Talent Search was in behavioral genetics, but I am going to describe some of my other research that is more mathematical. For a science project in my senior year, I was interested in branching out and exploring my newly-developed interest in physics. I had participated in a weekend program at the National Institute of Standards and Technology (NIST) in association with the 4-H called Adventure in Science. This program had weekly lecture/lab sessions to introduce students to different fields of science. After a few years, I started giving my own 3-hour lecture/lab session on Bacteriology and became friends with the organizers and head scientists of the program. I talked to Dr. Richard Steiner about finding a place to work over the school year. He invited me to work in his laboratory on the Electronic Kilogram Experiment.

My work originally served as a science fair project for the Intel International Science and Engineering Fair but turned into a longer-term project that truly contributed to the goals of the lab. My mentor, Dr. Steiner, gave me a list of possible projects, or smaller parts of the overall experiment. Since I did not have a background in circuitry, computer programming, or electrical engineering, I chose to work on finding a value for gravity more accurate than previously recorded. The project required me to learn how to approach instrumentation optimization, utilize new statistical tests, and apply some computer programming. One of the most exciting days of work occurred when I discovered that I could apply programming skills I acquired in my linear algebra class to my project. I needed a program that could graph in three dimensions, was easily manipulated, and used a color gradient to show height differences. MatLab, a matrix-based programming environment, fit that description perfectly. When I learned how to use MatLab in school, I didn't believe my teacher when she said the program had many uses and applications in the workplace. However, my knowledge proved useful and I was excited to apply it to something other than a practice problem in the classroom.

I am fortunate to live the in Washington, D.C. metropolitan area where I have access to many of the nation's research facilities. There are also many universities located in the area and technology corridors that provide a plethora of scientific research opportunities. But it's easy to get involved in research even if you live in a less populated area. Any community college or small university will have a professor who is willing to let you conduct research, especially if you are a volunteer and not a paid employee. However, you need to take math and science classes over the summer so you can load up on advanced classes during the year. Professors expect you to have a strong background in their area of research if you are going to work for them. Look online for research that others have done to identify a problem that builds upon previous work, or come up with something completely new! Never be afraid to try an experiment or ask a scientific question – it may turn out to have real applications and importance.

II. Research

Introduction and Background

The kilogram is the only base unit in the SI (International System/ Metric System) whose definition is based on a physical artifact instead of quantum references and fundamental properties of nature. The mass of the kilogram is currently based on the prototype platinum-iridium alloy kilogram standard in Paris, whose mass is believed to drift by about 50 parts per billion per century for an unknown reason. Sister prototype platinum-iridium alloys have changed mass over time due to chemical reactions with air. This variability in turn affects the values of other physical constants based on the mass standard. One of the constants dependent on the value of the kilogram is Planck's constant h, found from the Einstein and de Broglie equations:

$$E = mc^2$$

$$E = hv$$

$$h = mc^2 / v$$

The goal of the Electronic Kilogram Experiment is to improve the measurement of Planck's constant, providing an alternate definition to the current mass standard.

The Electronic Kilogram Experiment uses a watt balance system, which is a force balance that can act as an electric generator or as a motor. In one mode, current in the induction coils generates enough force to balance that of gravity on the reference mass, which will ultimately be defined as one kilogram. In a second mode, the coil is moved vertically while the velocity and induced voltage is measured. The watt balance switches between these two modes to obtain the ratio between mechanical power and electric power, which leads to finding Planck's constant h. The units of the mechanical/electrical power ratio should cancel out to one in the SI, assuming experimental error is minimal.

$$\frac{\textit{Mechanical Power}}{\textit{Electrical Power}} = \frac{Fv}{UI} \equiv 1 \qquad (1)$$

By replacing the power expressions with force (F) and current (I),

$$F = mg \qquad (2a)$$

$$I = U/Z \qquad (2b)$$

we get an equation with the mass (m) as the only factor not measured according to quantum standards with units that still reduce to one.

Figure 1. The watt balance.

Gravity (g) is a constant that can be measured locally. Velocity (v) is measured according to quantum electrical standards for length (light through a vacuum) and time (atomic clock). Voltage (U) and resistance (Z) are measured according to the Josephson effect and the Quantum Hall effect, respectively. Equation (1) now has measured variables:

$$\frac{mgv}{U\left(U/Z\right)} = \frac{mgv}{U^2/Z} \equiv 1 \qquad (3)$$

There are a number of theoretical relationships between Planck's constant, the Josephson effect constant (K), and the von Klitzing constant of the quantum Hall effect, and many other quantum numbers. This series of equations can be incorporated into equation (3) to get m and h in the same expression, shown simplified in equation (4).

$$\frac{mgv}{U^2/Z}\left(\frac{1}{K^2_{J-90}{}^{*}\ R_{k-90}}\right)=\frac{h}{4} \qquad (4)$$

The local acceleration of gravity (*g)* currently has the greatest uncertainty of all the terms in the equation, at 12 parts per billion. Reducing that uncertainty is the purpose of my project, as it will help provide a more accurate value for Planck's constant. I needed to find the value of gravity at a particular location in the watt balance about three meters high at the level of the test mass, which will is one kilogram. The upward force generated from the magnetic field in the watt balance must perfectly cancel out the force of gravity.

There was an absolute gravimeter in the same building as the watt balance. The gravimeter measures an absolute value for gravity at a particular location. Since the absolute meter cannot be disassembled and put inside the watt balance, I used a small gravity transfer meter to transfer the value of gravity from the absolute meter to the location in the watt balance. I also measured a gravity gradient at each of the two locations. A gravity gradient is the change in gravity with respect to the change in height. This was important because the exact vertical location of the mass may vary. I tested and characterized many of the transfer meter's internal correction and measurement factors with the hope of improving its future performance. Finally, I made gravity maps or surveys around the absolute meter and the watt balance to characterize those areas.

Instrumentation Studies

I tested four correction factors of the transfer meter: drift, temperature, tilt, and tidal correction. To correctly analyze the gravity data, it needed to be relatively quiet (stable temperature and tilt, small standard deviation, for example) and free of drift. The gravity readings of the small transfer meter showed a distinct downward linear drift. The transfer meter measures gravity by the extension of a fused quartz spring, which stretches over time and leads to drift. I looked at the internal and user-defined drift correction and after trial and error, I found the internal factor to be unrelated to the slope of the data and irrelevant. On the other hand, the user-defined correction factors reduced the drift slope as long as the machine was reset before use.

The data acquired from the gravity gradient and transfer measurements showed a drift that needed to be corrected before data analysis. The gravity data was analyzed graphically as gravity versus time, and best fit for linear or polynomial trends. To correct for the drift, the slope of the linear fit was subtracted from all data points, resulting in flat lines of gravity data. The difference between actual gravity and predicted gravity from the linear estimate was used to determine gravity variation between heights for the gradients and locations for the pier map.

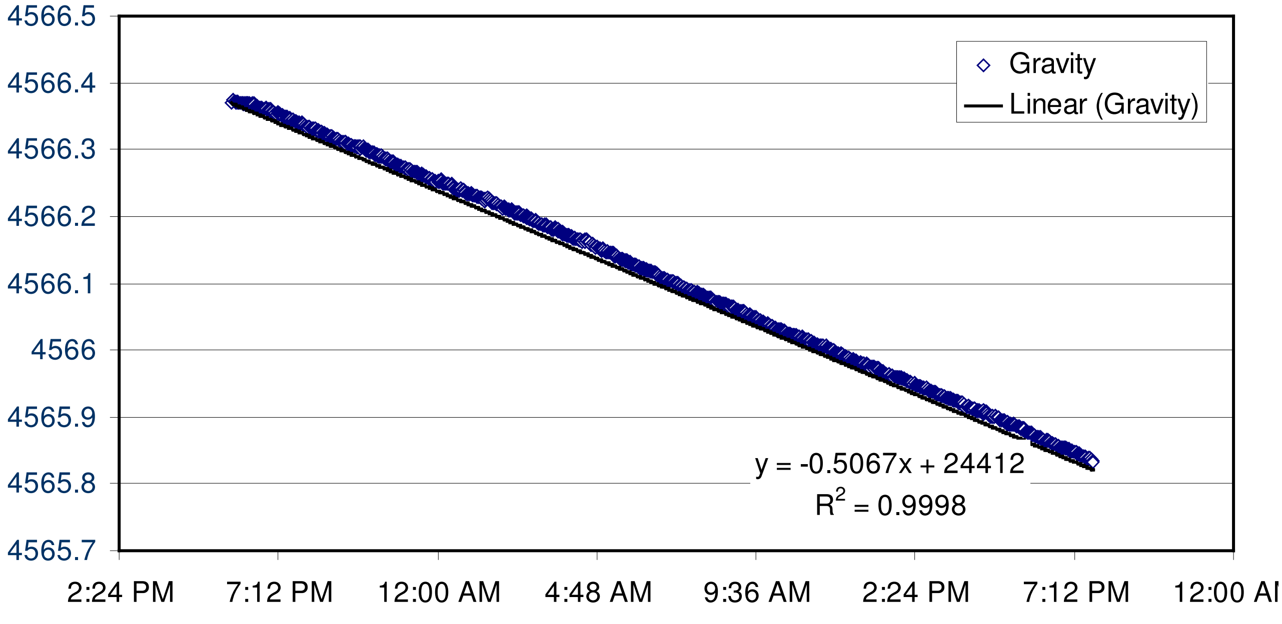

Figure 2. An example of gravity measurements taken over a 24-hour period showing a linear drift. The R^2 value of the linear estimate shows how closely the data fit a line. Gravity is shown in blue on the left axis, with units of milliGals (cm/sec^2).

I found two internal temperature regulators that indicated the actual ambient temperature and the change in temperature. The stabilization in temperature generally corresponded to a reduction in standard deviation in the gravity measurements. The temperature took approximately 4 to 5 hours to stabilize. Another factor measured by the gravity transfer meter was the tilt of the instrument on x and y axes. Since changes in angle affect the gravity measurement, an internal tilt correction is important to incorporate. The change in tilt was drastic over the first five hours of measurement periods, but then became fairly small for the rest of the acquisition period. The transfer meter sits on a metal tripod that may expand or contract with temperature. It was frequently transported between areas with different temperatures, and the stabilization of temperature correlates with the stabilization of tilt.

The transfer meter calculated tides corrections using an internal program that relies on the meter's relation to Greenwich Meridian Time (GMT), which is entered into the machine. When comparing transfer meter calculated tides with computer modeled tides, a phase shift was evident. After closer analysis and trial and error, a 10 hour shift was identified, due to an incorrect reference to GMT on the transfer meter. The operator settings had been set incorrectly because the user manual did not explain it clearly. Lesson learned: don't necessarily trust the manual or a machine's correction settings. Check them yourself!

Gravity gradients and transfer

The gradient was determined by dividing the average gravity difference between two points by the difference in height. The gravity gradient varies by $1/(R_E+r)^3$, with R_E as the radius from the center of the Earth. Since the height difference (r) in these gradients is so small compared to R_E, the gradient was treated as linear in r. I eventually measured three gradients: two on the first floor next to the absolute gravimeter and one on the second floor of the building in the watt balance. There was a slight difference in the gradients that was expected and reasonable. As the gravity measurements moved away from the center of the Earth, the gradients decreased, because gravity decreases moving away from Earth.

The gravity transfer was conducted by taking a gravity measurement at the absolute gravimeter, then one in the watt balance, then one back at the absolute gravimeter. The process was repeated approximately ten times in a row. The average difference between the two measurements was the correction factor. The value for the local acceleration of gravity in the watt balance can be found by adding the correction factor to the value from the absolute gravimeter.

Error Analysis

To determine a reasonable estimate for the combined relative standard uncertainty of the gravity transfer to the mass level in the watt balance, all the uncertainties from the measurements must be summed.

Uncertainty List

- Transfer meter measurements: 5 µGal
- Gradient location on 1st floor ≈ 0.2cm: (0.2*gradient) 0.63 µGal
- Random error from 1st floor gradient: 0.07 µGal/cm
- Level of test mass in watt balance ≈ 1 cm: (1*gradient) 3.15 µGal

- Random error from 2nd floor gradient: 0.05 µGal/cm
- Transfer between floors (st. dev. of transfer measurement): 2 µGal

The root sum squared equals 6.3 parts-per-billion, the total combined uncertainty (1 µG ~ 1 part per billion of earth's gravity),. The original uncertainty was 12 parts-per-billion. I aimed to reduce it to 10 parts-per-billion and was able to reduce it to almost 6 parts-per-billion.

Gravity Maps

I made two gravity maps: one around the absolute gravimeter and the other around the watt balance. Both maps were graphed on a coordinate grid using the meshgrid function of MATLab, a matrix-based programming environment. An example map set is shown in Figure 3. These maps allowed me to see patterns of gravity change over an area. Gravity appears to be higher near the gravimeter, caused by the concrete pier underneath it, than the rest of the room which has a regular foundation. At the north

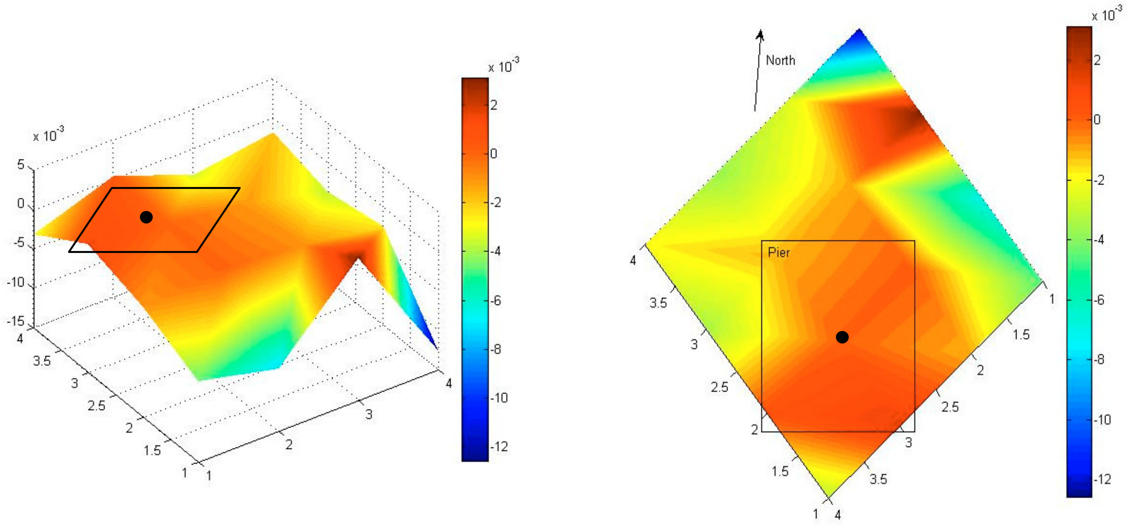

Figure 3. Side and top view of the 3-D gravity map of the area around the gravimeter.

end of the plot, there are two points that do not fit the expected pattern. This abnormality can be explained by the sensitivity of the transfer meter – as those two points were measured, someone walked by the door. This motion most likely caused a change in tilt, and thus, a change in gravity.

Earthquake Detection

On my last day of work, I looked at data from a 48-hour data acquisition trial and found that the standard deviation of the gravity measurements increased by a factor of three over a twenty minute period. The jump in standard deviation began at 08:52:11 on April 29, 2007. According to the United States Geological Survey records, a 6.2 Richter scale earthquake occurred in the Andreanof Islands of the Aleutian Islands with shock waves predicted to arrive in Washington, DC at 08:52:40 AM, April 29, 2007. The time of the standard deviation noise and the shock wave arrival correspond, implying that the transfer meter detected the vibrations from the earthquake.

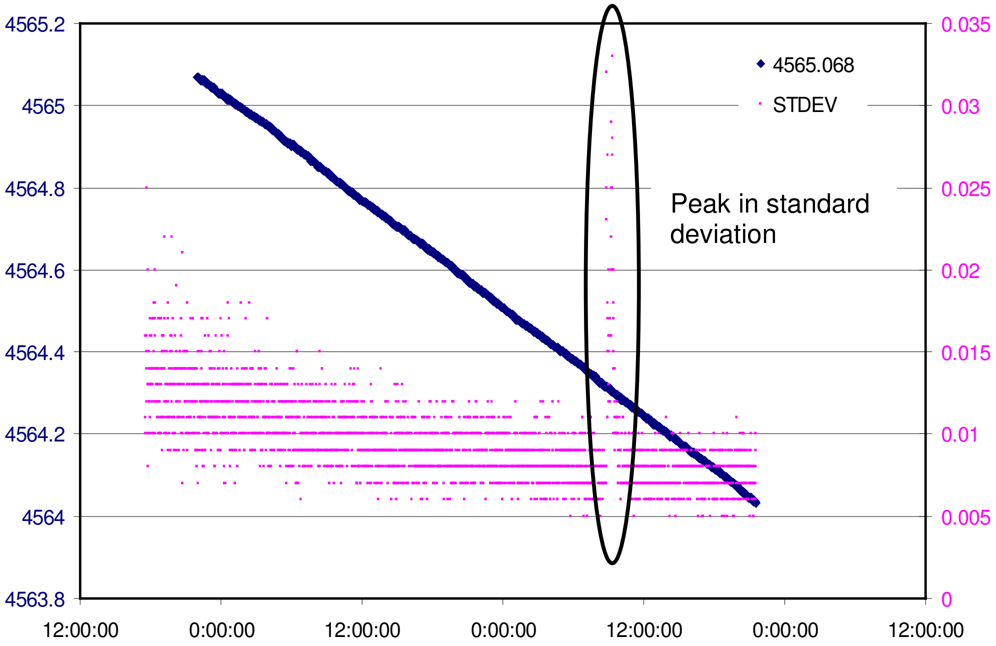

Figure 4. During a 48 hour trial, a sudden jump in standard deviation began at 8:52:11 AM on April 29, 2007. Gravity in mGal is shown on the blue axis while standard deviation is shown in pink.

Conclusions

The results of my study showed that the gravity transfer meter is sensitive enough to identify a change in gravity over a few millimeter height difference under present conditions. Tidal corrections and other factors must be monitored and tested on the transfer meter to make sure they are correct. The temperature and tilt changes level out over time, and less change reduces the standard deviation of the gravity data. The drift is down-sloping and linear, and transfer meter internal drift reduction factors only function when the instrument is reset.

Gravity gradients may be treated as linearly varying, depending on the height and surrounding environment. The gravity maps show that the meter detected a difference between the concrete pier and the rest of the room, shown by the difference in gravity. Finally, we have not yet reached the lowest limit of the combined uncertainty for the acceleration of gravity.

References

Flamsted, S. 2007. GRACE in Space: A Pair of Satellites Map Subtle Variations in Earth's Gravitational Field, Revealing Secret Craters, Undersea Mountains, and the Impact of Climate Change. *Discover* 28 (3): 44-48.

The Electronic Kilogram. n.d. *Quantum Electrical Metrology.* Retrieved February 5, 2007, from http://www.eeel.nist.gov/817/whatwedo-817g/kilogram/elec-kilo.html

Smith, A.B., Walker, J.P., Western, A.W., Ellett, K.M. 2005. Using ground-based gravity measurements to monitor changes in terrestrial water storage. *Engineers Australia 29th Hydrology and Water Resources Symposium.* 21-23 February, Canberra.

Steiner, R.L, Williams, E.R, Liu, R, Newell, D.B. 2007. Uncertainty improvements of the NIST Electronic Kilogram. *IEEE Transactions on Instrumentation and Measurement.* 56 (2): 592-596.

Steiner, R.L, Newell, D.B, Williams, E.R. 2005. Details of the 1998 Watt Balance Experiment Determining the Planck constant. *J. Res. Natl. Inst. Stand. Technol.* 110 (2005): 1-26.

Steiner, R.L, Williams, E.R, Newell, D.B, Liu, R. 2005. Towards an electronic kilogram: an improved measurement of the Planck constant and electron mass. *Metrologia* 42 (2005): 431-441.