# Assessing the Accuracy of an Analytical Method to Determine the Shape of Rotating Neutron Stars

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***(i) Personal Account***

Astronomy has always captivated me, and after being introduced to the constellation Orion at a young age, I marveled at its steadfast loyalty as it returned each winter. The night sky seemed awesome and mystifying, and was so far away from the violent actions of humans I saw strewn across televisions, magazines, and newspapers. I began researching and following the patterns of the moon and the night sky, as well as lunar and solar eclipses. I was thrilled when our family got a telescope, enabling me to view more closely the objects with which I was enthralled. At the end of $9^{th}$ grade, I applied for our high school's science research program, and somewhat naively began plunging the depths of the immense field of astronomy. Upon reading papers in a scientific and mathematical language that I did not understand, I realized that I would have to teach myself the basic concepts. I searched websites and books, and purchased a beginner's astronomy textbook. After looking into the many specific applications of astronomy, I decided to study pulsars, largely misunderstood, mind-bogglingly dense, light-emitting, collapsed stars.

It was at first extremely frustrating to plow through the literature on the subject of pulsars, which seemed as dense as the pulsars themselves. However, my curiosity trumped my own doubts and the words of those who told me I could not conduct research in astronomy due to its inherent difficulty. I obtained definitions of perplexing terms and equations in my reading, and taught myself some basic physics concepts. Despite having taken calculus, I had not taken a class in physics, and I could not have done the work for this project on my own. I received a tremendous amount of help from others, and I attribute much of my success to my mentor, Doctor James Lattimer of the State

University of New York at Stony Brook, whom I contacted towards the end of my sophomore year of high school. Dr. Lattimer generously gave his time to help me formulate a research project in an area of science about which I knew virtually nothing and taught me the basics (if they can be called basics) of the advanced mathematics and computer programming language necessary to complete my project. Because of encouragement from Dr. Lattimer, I applied for the Simons Summer Research Fellowship at Stony Brook, without which I would not have been able to complete my research. This fellowship allowed me to work during the week from nine to five on my project, and introduced me to the sometimes tedious and challenging, yet exciting life of a researcher. It was a rewarding experience simply to be surrounded by other scientists sharing dizzying ideas over Thursday coffee breaks. I came back from the program with an overwhelming abundance of new knowledge, mounds of data to analyze, and an ability to write computer programs in a seemingly ancient programming language. Spending a large part of my summer doing scientific research allowed me to accomplish what was necessary to write my Intel paper successfully.

I would not have been able to accomplish anything without the guidance of the outstanding science research program offered by my school, and of course my ever-supportive parents, sisters, and friends. The Ossining High School Science Research Program is unique in that members of the program are more a family than simply a group of students and teachers striving to do some research. My fun, quirky, science research teachers and advisors, that praiseworthy powerhouse pair Mr. Angelo Piccirillo and Ms. Valerie Holmes, have made my time at high school unforgettable. They never fail to motivate and amaze me every day, and are always there to provide support, like listening to a presentation for the hundredth time, cheering at competitions, or simply helping a frantic student de-stress over a cup of coffee in that tiny hole-in-the-wall science research room I call home. They give of themselves relentlessly and tirelessly, as though each and every science research student is their own child.

My advice to fellow student researchers is this: even if your school does not offer a science research program, do not let that stop you! Completing the daunting task of a research project is difficult, but is much more rewarding if you surround yourself with supportive people, whether it be researchers, parents, teachers, or friends. Do not be afraid to reach out to others for help, because building a great research experience is not always about whether or not your project worked. Probably the most important skill I have learned from my time doing scientific research is communication—through telephone calls, emails, power points, poster presentations, or face-to-face discussions. Especially in mathematics projects, where concepts can be difficult and often purely theoretical, it is important to consult the experts even when intimidated. Obtain feedback every step of the way, because different people will offer diverse and valuable comments. I have learned to look both a younger classmate, and a college physics professor in the eye and say, "I have no idea how to do this. Can you help me?" Get past your initial reluctance, as I did (and sometimes still have to do), and admit to yourself that as only a student, it is ok to ask questions, because in the end, you will gain so much more if you do.

***(ii) Research Report***

***I. Introduction***

A neutron star is one of the densest objects in the universe, containing on average about 1.5 solar masses and having a radius of approximately 12 kilometers (Lattimer et al., 2004). For this reason, the neutron star may exhibit unique particle phenomena including superfluidity, superconductivity, and hyperon and quark-dominated matter, and provides many opportunities to study and test the theories of particle, nuclear, and dense matter astrophysics (Manchester et al., 2004). Despite the many significant advances in the field, there is still much that remains unknown regarding neutron stars, including their radii and shapes as they rotate (Shapiro et al., 2004). It is imperative that these properties be established before astronomers can use neutron stars to test more complex aspects of physics (Webb et al., 2007).

A neutron star is formed when a massive star undergoes a Type II, or core-collapse supernova (Lattimer et al., 2004). Nucleons within the core collide and rebound, sending out a powerful shock wave which expels the outer layers of the star. The inner layers then collapse under the star's gravity, causing the protons and electrons in the inner layer to combine and form neutrons (Manchester et al., 2007). The star's rapid rotation is generated by angular momentum that is conserved in the supernova collapse and can be increased at a later time due to mass accreted from a companion star. When a neutron star emits electromagnetic radiation that is detectable from Earth, it is called a pulsar. Beams of radiation emitted by two

"The Lighthouse Effect"

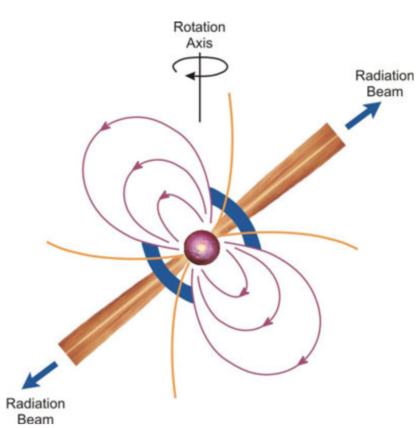


Figure 1. National Radio Astronomy Observatory

opposite "hot spots" on the surface of the pulsar sweep around as it rotates, creating what is known as a "lighthouse effect" (Manchester et al., 2007) (Figure 1). When these beams pass in the direction of Earth, they can be detected as a series of pulses.

The period of resolution or amount of time between each recorded pulse is extremely constant, and can vary from about a few milliseconds to approximately a second. For this reason, pulsars are ideal objects for testing theories of neutron stars (Manchester et al., 2004). The pulse patterns, however, are not always simple to interpret because they are affected by the non-spherical shapes of highly rotating stars. An understanding of the shapes of rotating neutron stars could better explain pulse patterns and the possible ways in which variations of patterns occur (Lattimer et al., 2005).

If the underlying equation of state (EOS) relating the density and pressure of matter were known, the general relativistic equation of stellar structure could be exactly solved to determine the rotating neutron star's size and shape. The shape determination, however, requires a complex numerical evaluation including gravitation and general relativity (Zdunik et al., 2004, Haensel et al., 1999). These equations are too time-consuming to be used to assess observational data. An analytical formula, if accurate enough, would make these comparisons more straightforward. Previous research (Lattimer et al., 2005) has suggested that a basic formula does exist, but such a formula has yet to be tested to determine its accuracy and applicability across a broad range of equations of state and rotation rates.

## *II. Purpose/Hypothesis*

The purpose of this project is to assess the accuracy of relatively basic structural formulae derived from a simplified Newtonian model that relates neutron star spin frequency, mass, polar and equatorial radii, and shape. We hypothesize that data produced by our analytical method will be relatively accurate compared to that of the previous studies in assessing shape, but will require some further algebraic manipulation to increase accuracy.

***III. Methods***

*3.1 The Roche Model*

The following equations are adapted from the Newtonian Roche model for uniformly rotating stars, as outlined in Shapiro, 2004. The Roche model assumes that the star's gravity is the same as for an object in which the mass is concentrated in its center. Three major forces are in action as this celestial body rotates: gravity, which pulls the body inward, and pressure gradient and centrifugal forces, which push outward. These forces are in a balance known as hydrostatic equilibrium (Lattimer et al., 2004). The equation of hydrostatic equilibrium of a rotating object in Newtonian gravity is given by:

$$\frac{\nabla P}{p} = -\nabla(\Phi_G + \Phi_c) \qquad (1)$$

where P and *p* are the pressure and density of matter, $\Phi_G = -GM/r$ is the Newtonian gravitational potential assuming a concentration of mass (M) in the center of the star and distance from the origin *r*. The symbol $\nabla$ denotes the gradient operator, which in one dimension would simply be $\frac{d}{dx}$. The centrifugal potential is $\Phi_c = -\frac{1}{2}\Omega^2 r_p^{\ 2} \sin^2\theta$, where $\theta$ is the angle measured from the pole of the rotating star and $\Omega$ is the angular frequency. When this equation is integrated, one obtains the Bernoulli integral:

$$h + \Phi_G + \Phi_c = H \qquad (2)$$

where $h$ is the thermodynamic quantity known as the enthalpy at zero temperature, and $H$ is the constant of integration. Formally, $h = \int \frac{\Delta P}{p}$ and is defined to be zero at the star's surface. When this equation is evaluated at the pole of the rotating object, where $\theta$ and therefore $\Phi_c$ and $h$ are zero, the constant of integration is seen to be $H = \frac{GM}{R_p}$. Here, $G$ is Newton's gravitational constant, $M$ is the mass of the celestial body, and $R_p$ is the radius at the object's pole. Detailed numerical calculations suggest that to a reasonable approximation, $R_p$ remains nearly constant as the rotation rate $\Omega$ is increased (Papaloizou & Whelan, 1973).

*3.2 The Maximum Case*

As a rapidly rotating celestial object spins, it begins to flatten, increasing its equatorial radius (Salgado et al., 1994). The rotating body breaks apart at the mass-shedding limit where the gravitational and centrifugal forces can no longer be balanced. This occurs when the orbital velocity at the equator is equal to the surface velocity. The maximum spin frequency for a rotating star before it breaks apart is given by the following equation derived from Kepler's laws of orbital motion:

$$\Omega_k = \sqrt{\frac{GM}{\left(R_{eq_{\max}}\right)^3}} \tag{3}$$

Here, $R_{eq_{\max}}$ is the equatorial radius of the maximally rotating body.

When equation (3) is substituted into the Bernoulli integral (2) assuming $R_p$ is unaffected by rotation, one obtains the following formula:

$$R_{\mathrm{eq}_{\max}} = \frac{3R_p}{2} \quad (4)$$

This suggests that in order for an object to remain intact while rotating, the radius at the equator, $R_{eq}$, must be less than or equal to 1.5 times the radius at the object's pole, $R_p$ (Cook et al., 1993). Using the results for maximum spin rate, equation (2) can be used again to obtain the following equation:

$$1 - \frac{R(\theta)}{R_p} + \frac{4}{27}\left(\frac{R(\theta)}{R_p}\right)^3 = 0 \quad (5)$$

Equation (5) has the cubic solution of the following equation:

$$R(\theta) = \frac{R_p 3\sin\left(\theta/3\right)}{\sin(\theta)} \quad (6)$$

Here, $R_p$ is the polar radius, and $R(\theta)$ is the stellar radius at angle θ from the pole. If equations (3) and (4) are determined to be accurate, equations (5) and (6) can then be utilized for the purposes of estimating the shape of the maximally rotating neutron star.

*3.3 The Intermediate Case*

For stars rotating at a frequency less than the maximum, the following formula is derived from equation (2), evaluated at the equator:

$$\frac{\Omega^2 R_p{}^3}{2GM} = \left(\frac{R_p}{R_{eq}}\right)^2\left(1 - \frac{R_p}{R_{eq}}\right) \quad (6)$$

It is convenient to employ the quantity known as eccentricity, defined by

$$e = \left(1 - \frac{r_p^{\ 2}}{r_{eq}^{\ 2}}\right)^{\frac{1}{2}} \qquad (7)$$

to describe intermediate cases. Note that $e = 0$ for a non-rotating body, and the maximum value of the eccentricity is $e_{\max} = \frac{\sqrt{5}}{3}$.

*3.4 Interactive Data Language*

The predictions of equations (3), (6), and (7) were evaluated in the program Interactive Data Language (IDL), Version 7.0. Relating neutron star data were obtained from the following previous studies:

i. *Cook, G.B., Shapiro, S.L., Teukolsy, S.A., "Rapidly Rotating Neutron Stars in General Relativity: Realistic Equations of State" (1994):* Cook et al. have constructed realistic rotating neutron star model sequences in general relativity using fourteen realistic EOS. Rotation rates, equatorial radius, eccentricity, and mass values were obtained from tables 12-23 of Appendix C. The gravitational constant $G=6.667\times10^{-8} g^{-1} cm^3 s^{-2}$ and solar mass $M_\circ=1.989\times10^{33}$g.

ii. *Cook, G.B., Shapiro, S.L., Teukolsy, S.A., "Rapidly Rotating Polytropes in General Relativity" (1994):* Here, polytropic EOS are used to construct rotating neutron star model sequences in general relativity. Polytropic equations assume that $P = k\rho^{n+1/n}$, where $P$ is pressure, $\rho$ is density, $k$ is a constant and $n$ is known as the polytropic index. Polytropic models are not considered realistic, but are useful tools for modeling neutron stars. Rotation rates, equatorial radius, eccentricity, and mass values were obtained from tables 7-21 of the Appendix.

iii. *Friedman, J.L., Ipser, J.R., Parker, L., "Rapidly Rotating Neutron Star Models" (1986):* Twelve EOS were used by Friedman et al. in this study to model sequences of neutron stars at various fixed injection energies. Rotation rates, equatorial radius, eccentricity, and mass values were obtained from tables 3-11. The gravitational constant $G=6.670\times10^{-8}g^{-1}cm^{3}s^{-2}$ and solar mass $M_{\circ}=1.987\times10^{33}g$.

iv. *Salgado, M. Bonazzola, S., Gourgoulhon, E., Haensel, P., "High Precision Rotating Neutron Star Models: I. Analysis of Neutron Star Properties" (1994); Salgado, M. Bonazzola, S., Gourgoulhon, E., Haensel, P., "High Precision Rotating Neutron Star Models: II. Large Sample of Neutron Star Properties":* Twelve realistic, as well as two polytropic EOS, were used to produce an extensive sample of over two thousand neutron star models, which were obtained electronically through the astronomical databases stored at the Centre de Données Astronomiques de Strasbourg. Rotation rates, equatorial radius, and mass values were taken from models, were grouped by EOS, and split further according to baryon mass values. Because there were no eccentricities given for these models, we could not calculate the polar radius from the equatorial radius. For Salgado et al., II, however, within each group of set baryon mass, the rotation rates increased from zero to the maximum. We were thus able to assume the value of the star's polar radius to be the same as its nonrotating equatorial radius, which was then held constant as the polar radius for each model in a set. The gravitational constant $G=6.670\times10^{-11}m^{3}kg^{-1}s^{-2}$ and solar mass $M_{\circ}=2.0\times10^{33}g$.

Maximum velocities were calculated with obtained equatorial radii and masses using equation (3) and compared to that of the previous studies. Polar radius values were calculated in equation (7) using equatorial radius values, and maximum equatorial and polar radii were compared to one another based on the criteria of equation (4). For intermediate cases, both sides of equation (6) were calculated separately using mass and radii values and compared to one another. The data were analyzed in IDL to determine percent error between calculated and obtained previous results, as well as to create appropriate linear regression models relating our study to previous studies.

***IV. Results***

It was determined that our results for maximum rotational frequency deviated by an average of 6.867% from that of the previous studies, while the intermediate equations had a larger average margin of 23.253% error. For this reason, the intermediate data were also broken down according to EOS used by the previous studies in calculation. The calculated ratios of polar to equatorial radii had an average percent error of 9.812% when compared to the original 3:2 ratio.

*4.1 Cook et al., 1994: Realistic EOS*

Even as the equatorial radius of the star increased, the ratio between our results and Cook's results remained relatively constant with a mean quotient of 1.01982 excluding outliers (Figure 2). Thus, the maximum spin data had an average error of 1.098% when compared to that of the previous study. The ratios between polar and equatorial radii of maximally rotating cases confirm the accuracy of equation (4) within an average of 9.6032% of the original 3:2 polar to equatorial radius ratio. As equatorial radius increases, there is only a slight increase in the ratio (Figure 3).

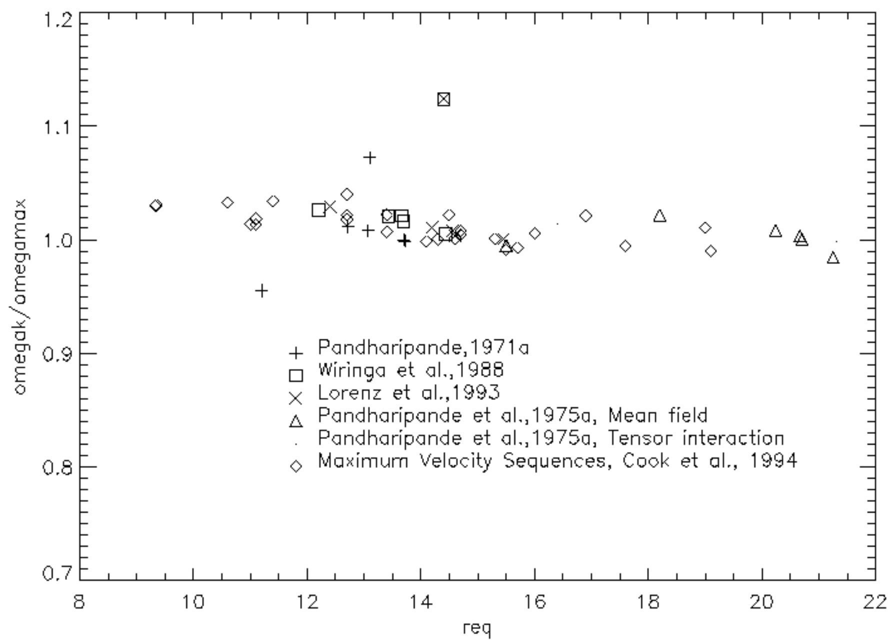


Figure 2. Comparison of equatorial radius and ratio of maximum rotation rate calculated in equation (3) to Cook et al.'s maximum rotation values. Various symbols represent equations of state.

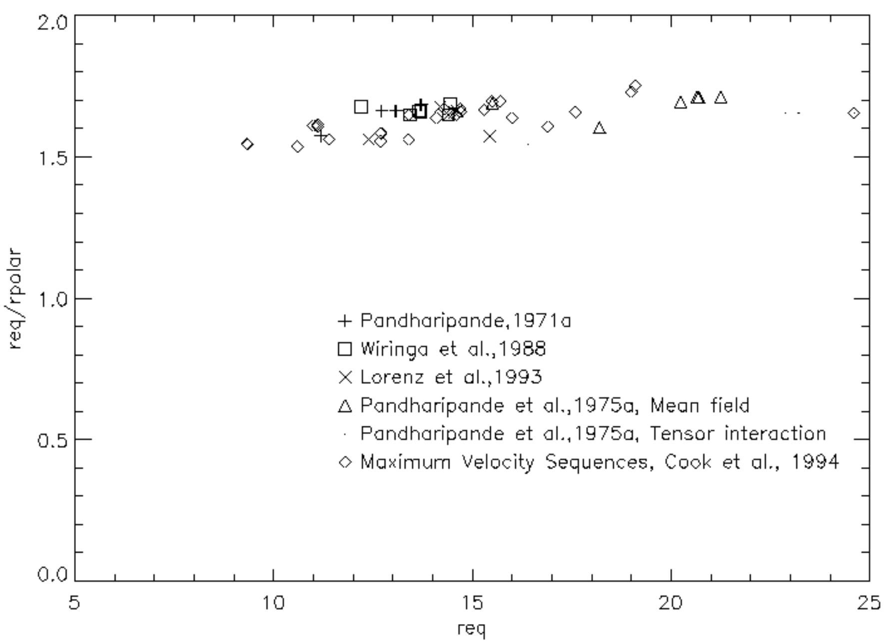


Figure 3. Comparison of equatorial radius to polar-equatorial radii ratio based upon equation (4).

The two sides of intermediate equation (6) were found to have an average deviation of 25.296% from one another (Figure 4), suggesting that the equation is somewhat accurate, but that a numerical constant could be multiplied by the right side of equation (6) to increase accuracy.

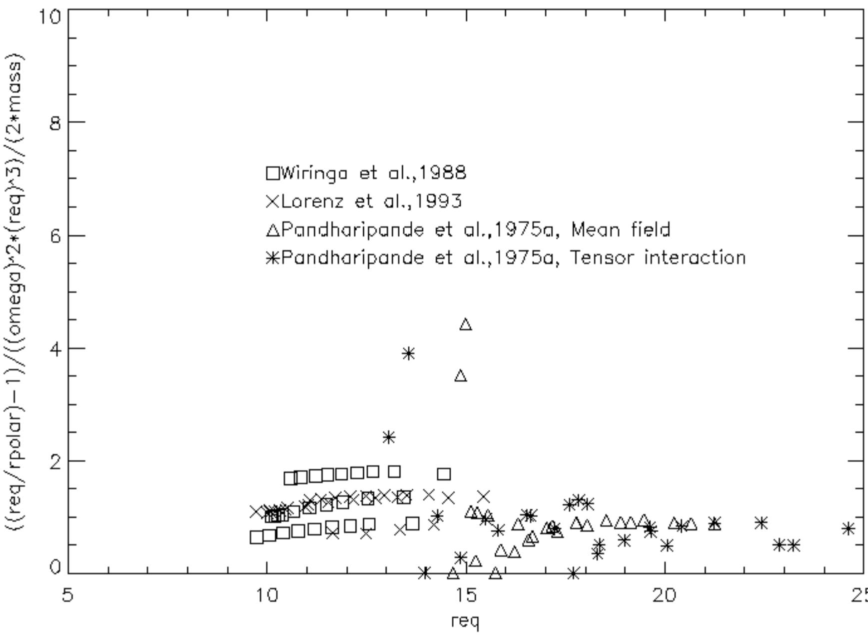


Figure 4. Comparison of equatorial radius to ratio of left and right sides of intermediate rotation rate equation (6) calculated using Cook et al.'s mass and radius values.

Linear associations were performed between each of the sets of variables. The correlations and coefficients of determination approached 1.000, with *r*-values 0.980, 0.998, and 0.994, and $r^2$ values 0.961, 0.996, and 0.987 for intermediate cases, maximum cases, and equatorial-polar radius ratios, respectively. In addition, slope (*b*) values 1.336 and 1.037 approached 1.000 for intermediate and maximum equations, respectively, and 1.741 approached the 3:2 ratio assumed by equation (4). These values (Table 1) all demonstrate that our methods hold somewhat true for the values produced by Cook et al.

Table 1. Correlation (*r*), coefficient of determination ($r^2$), and slope (*b*) and significance of the linear regression line (*sig.*) for left compared to right sides of intermediate rotation equation (6), maximum rotation values (3) compared to Cook et al. values, and ratio of polar to equatorial radius (4).

| Equation | *R* | $r^2$ | *b* | *sig.* |
|---|---|---|---|---|
| Intermediate (6) | 0.980 | 0.961 | 1.336 | 0.038* |
| Maximum (3) | 0.998 | 0.996 | 1.037 | 0.021* |
| Equatorial/Polar Radius (4) | 0.994 | 0.987 | 1.741 | 0.002* |

*p-value <0.05

*4.2 Cook et al., 1994: Polytropic EOS*

The polytropic EOS were slightly less accurate than the realistic EOS for maximum rotation equation (3), producing spin frequencies within an average of 8.1299% from obtained spin frequency data (Figure 5).

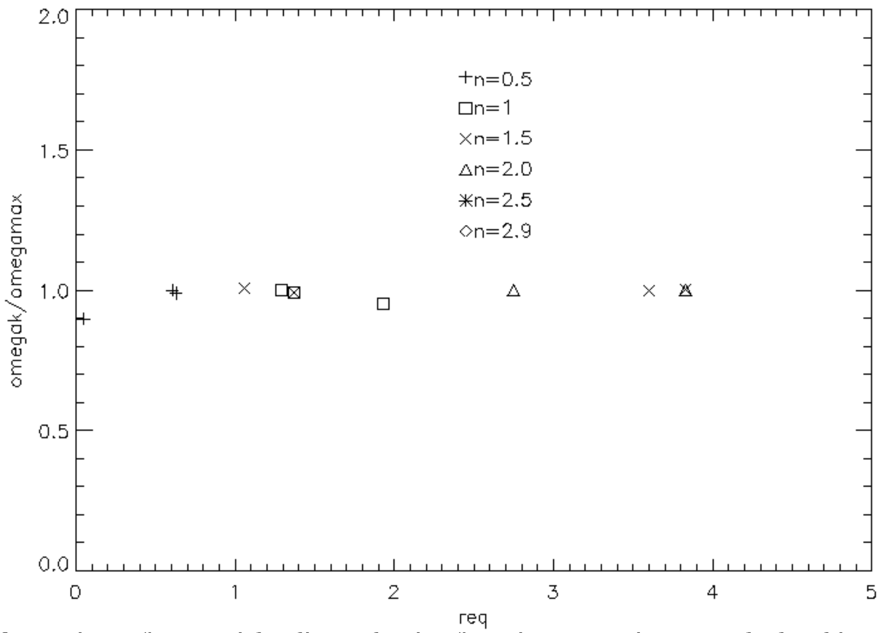


Figure 5. Comparison of equatorial radius and ratio of maximum rotation rate calculated in equation (3) to Cook et al.'s maximum rotation values for polytropic EOS.

For maximally rotating polytropic EOS, the ratio between polar and equatorial radius had a percent error of 7.663% when compared to the predicted 3:2 ratio, and decreased slightly as equatorial radius increased (Figure 6).

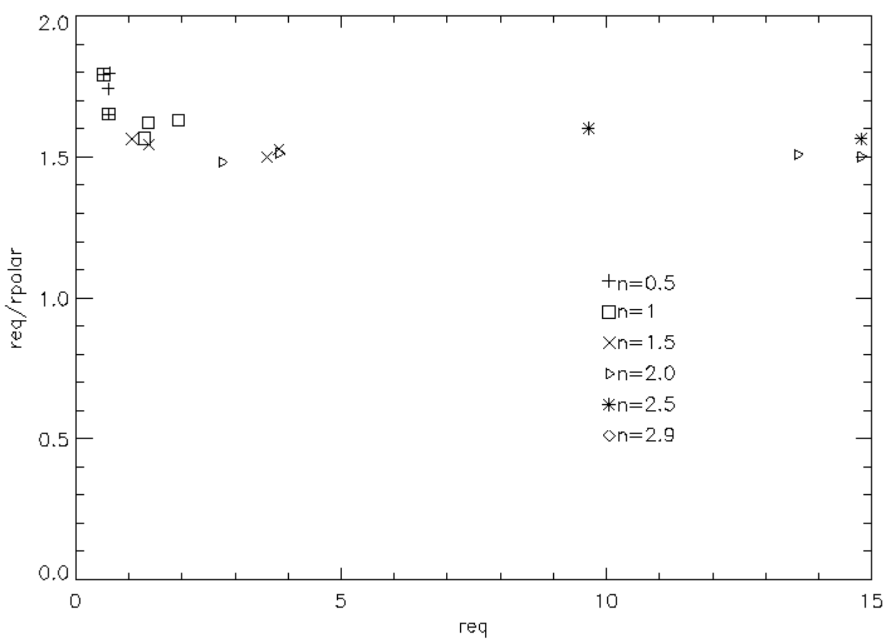


Figure 6. Comparison of Cook's polytropic equatorial radius to polar to equatorial radii ratio based upon equation (4)

For intermediate cases, the ratio between the two sides of equation (6) had a mean

The left side of intermediate equation (6) had a percent error of 18.908% when compared with the right side of the equation, excluding outliers (Figure 7).

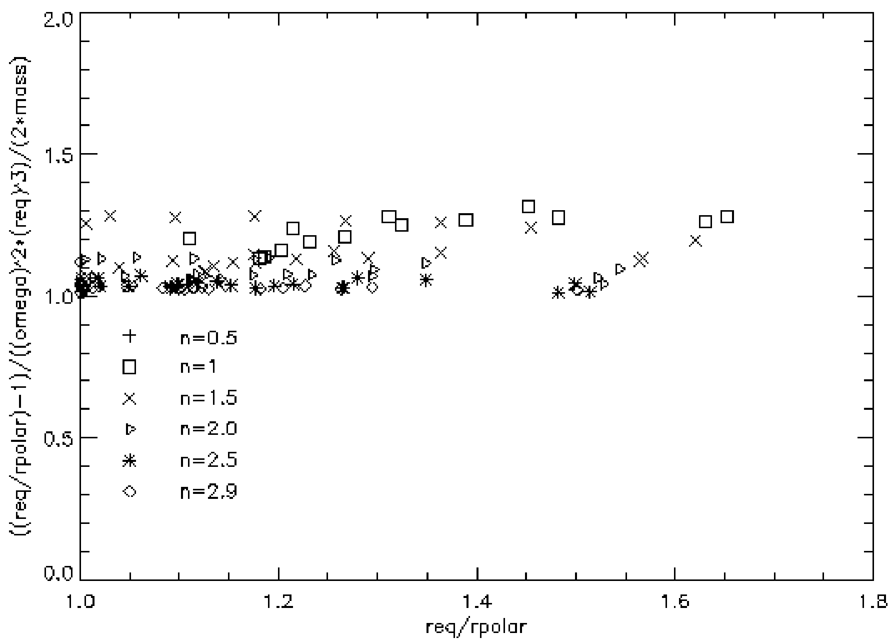


Figure 7. Comparison of equatorial radius and ratio of intermediate rotation rate calculated in equation (6) to Cook et al.'s rotation values for polytropic EOS.

The results from linear regression models for Cook et al., polytropic EOS were less strongly correlated than Cook et al., realistic EOS for intermediate spin frequencies, but greater significance for maximum rotation and ratio of equatorial to polar radius: 0.090 compared to 0.042 and 0.001, respectively (Table 2).

Table 2. Correlation (*r*), coefficient of determination ($r^2$), and slope (*b*) and significance of the linear regression line (*sig*) for left compared to right sides of intermediate rotation equation (6), maximum rotation values (3) compared to Cook Polytrope et al. values, and ratio of polar to equatorial radius (4).

| Equation | *r* | $r^2$ | *b* | *Sig.* |
|---|---|---|---|---|
| Intermediate (6) | 0.941 | 0.885 | 1.342 | 0.090** |
| Maximum (3) | 0.999 | 0.999 | 1.011 | 0.042* |
| Equatorial/Polar Radius (4) | 0.999 | 0.999 | 1.502 | 0.001* |

*p-value <0.05, **value approaches statistical significance

*4.2 Friedman et al., 1986*

The ratio between maximum obtained and calculated frequencies remained relatively constant as equatorial radius increased (Figure 8), with a mean of 12.353%

error. The equatorial to polar radius ratio had an average of 7.662% error compared to the predicted 3:2 ratio (Figure 9).

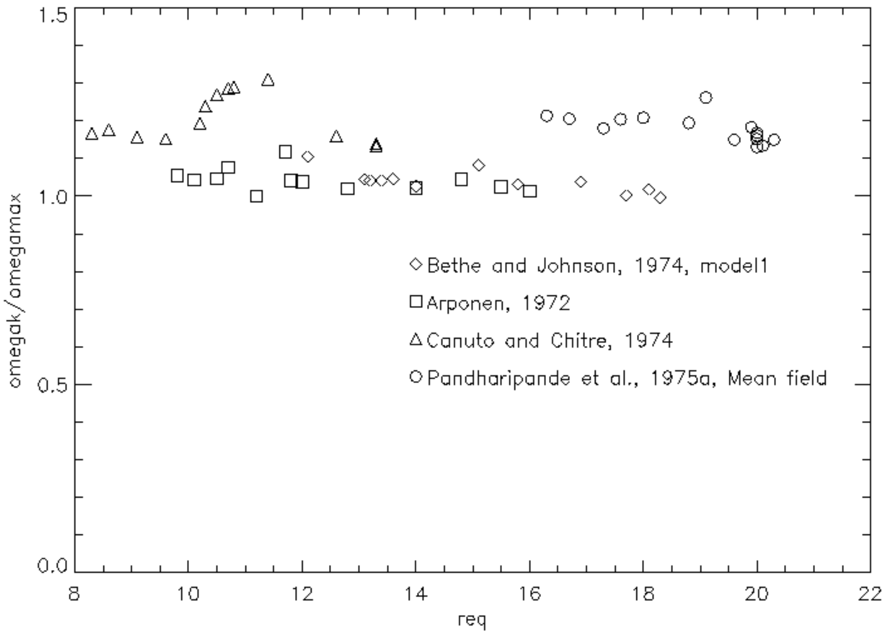


Figure 8. Comparison of equatorial radius and ratio of maximum rotation rate calculated in equation (3) to Friedman et al.'s maximum rotation values for polytropic EOS.

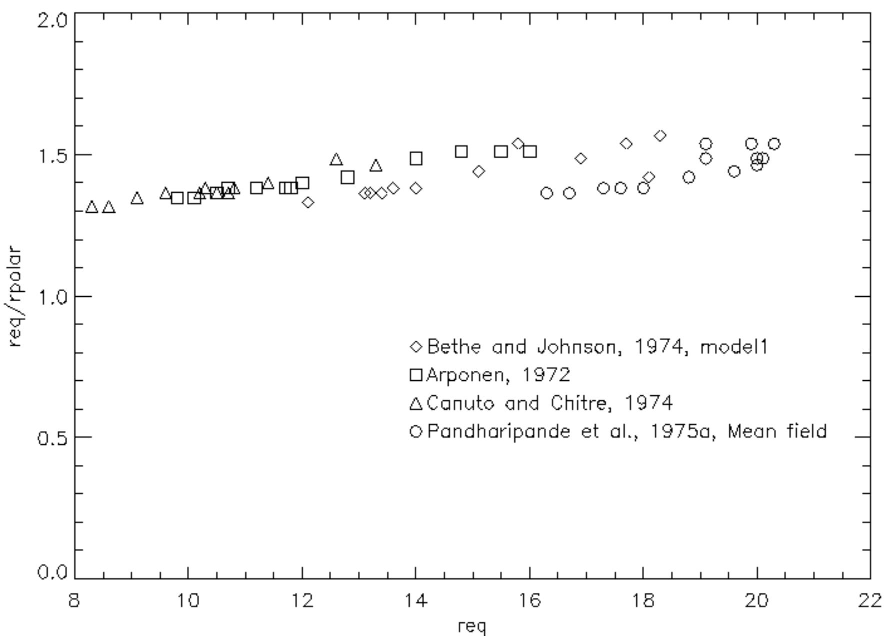


Figure 9. Comparison of Frieman et al.'s equatorial radius and polar to equatorial radii ratio based upon equation (4)

For intermediate cases, the left and right sides of equation (6) deviated 9.297% from one another (Figure 10).

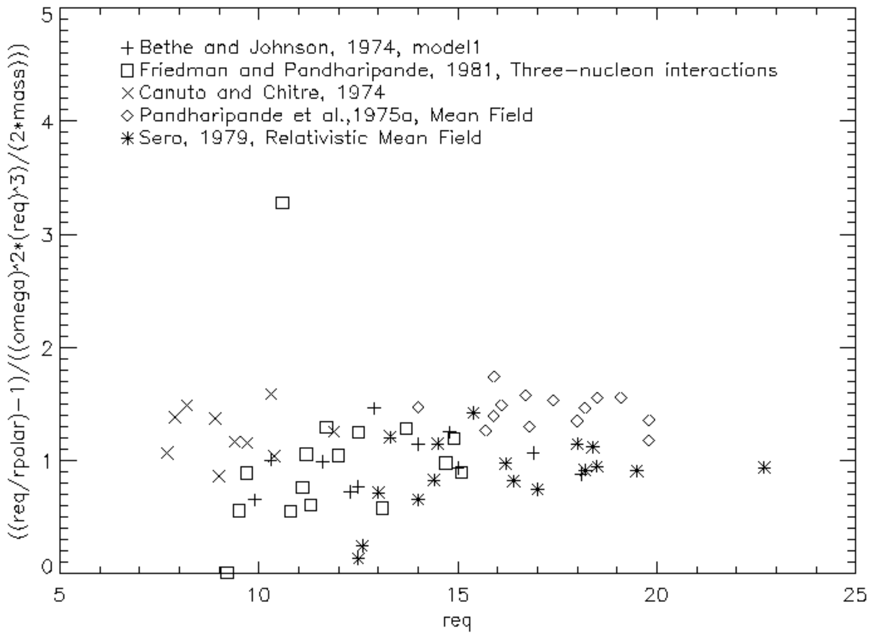


Figure 10. Comparison of equatorial radius with ratio of intermediate rotation rate calculated in equation (6) to Friedman et al.'s rotation values.

The linear model for values from Friedman et al. compared to our calculated values (Table 3) demonstrates the relative accuracy of our formulae, as well as higher degree of accuracy of the maximum equations as compared to that of the intermediate cases.

Table 3. Correlation (*r*), coefficient of determination ($r^2$), and slope (*b*) and significance of the linear regression line (*sig*) for left compared to right sides of intermediate rotation equation (6), maximum rotation values (3) compared to Friedman et al.'s values, and ratio of polar to equatorial radius (4).

| Equation | *r* | $r^2$ | *b* | *sig.* |
|---|---|---|---|---|
| Intermediate (6) | 0.955 | 0.913 | 1.042 | 0.162 |
| Maximum (3) | 0.980 | 0.961 | 1.084 | 0.030* |
| Equatorial/Polar Radius (4) | 0.987 | 0.973 | 1.592 | 0.001* |

*p-value <0.05

*4.4 Salgado et al., 1994 I.*

Obtained rotational frequency values had a mean of 11.337% error for the maximum case (Figure 11), and 23.536% for the intermediate case (Figure 12). The ratio of radii could not be calculated, as polar radius was unable to be calculated.

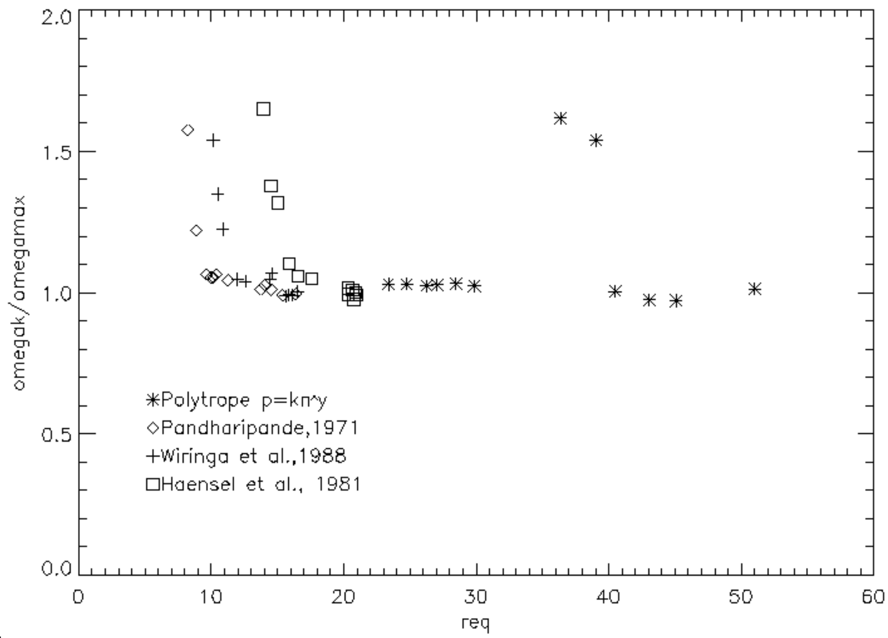


Figure 11. Comparison of equatorial radius and ratio of maximum rotation rate calculated in equation (3) to Salgado et al.'s maximum rotation values for various EOS.

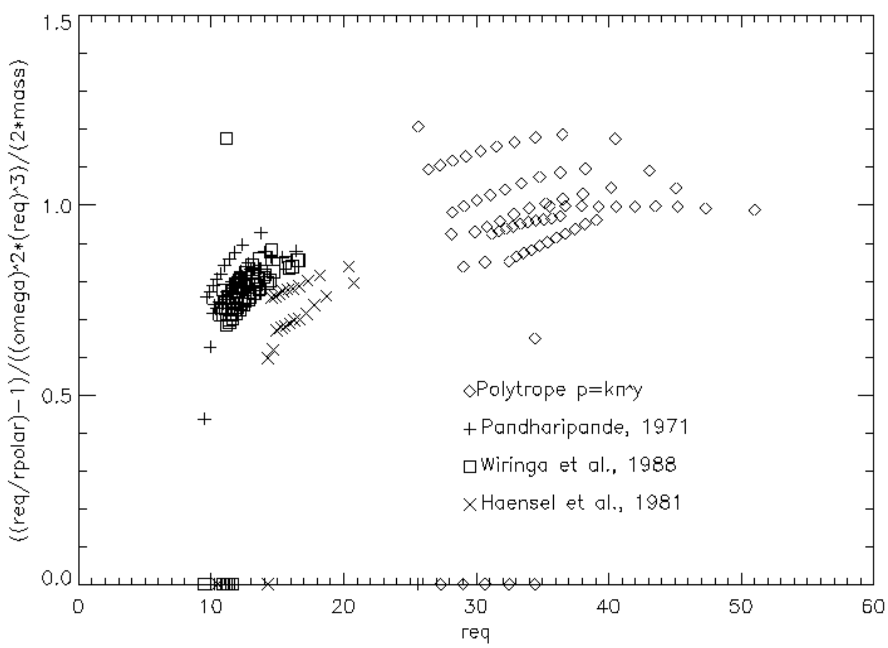


Figure 12. Comparison of ratio of equatorial radius to polar radius with ratio of intermediate rotation rate calculated in equation (6) to Salgado et al.'s rotation values.

The linear regression analysis interestingly produced more significant results for intermediate cases than for maximum (Table 4), despite the fact that the mean percent errors were greater for intermediate than for maximum cases, likely due to the presence of more extreme outliers in Figure 11 than in Figure 12.

Table 4. Correlation (*r*), coefficient of determination ($r^2$), and slope (*b*) and significance of the linear regression line (*sig*) for left compared to right sides of intermediate rotation equation (6) and maximum rotation values (3) compared to Salgado I et al.'s values.

| Equation | *r* | $r^2$ | *b* | *sig.* |
|---|---|---|---|---|
| Intermediate (6) | 0.980 | 0.961 | 0.896 | 0.029* |
| Maximum (3) | 0.955 | 0.912 | 0.659 | 0.329 |

*p-value <0.05

### *4.5 Salgado et al., 1994 II.*

Our calculated maximum spin frequency values deviated from those calculated by Salgado et al. II with a mean error of 1.4187% (Figure 13), while the ratio of equatorial to polar radius had a mean 14.321% error (Figure 14). The graph for intermediate ratio is omitted, as this extensive list of over two thousand data values produced varying results, with an average of 39.227% error.

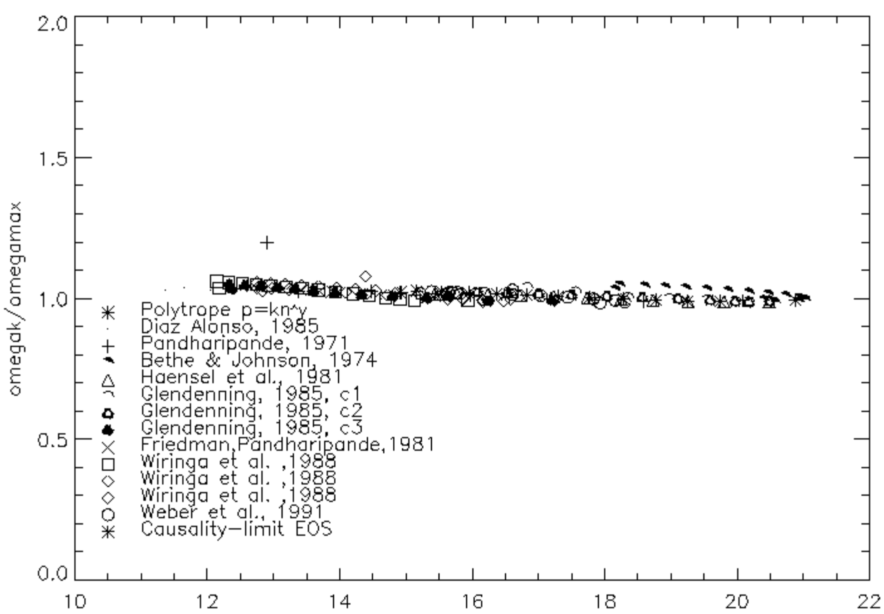


Figure 13. Comparison of equatorial radius and ratio of maximum rotation rate calculated in equation (3) to Salgado II et al.'s maximum rotation values for various EOS.

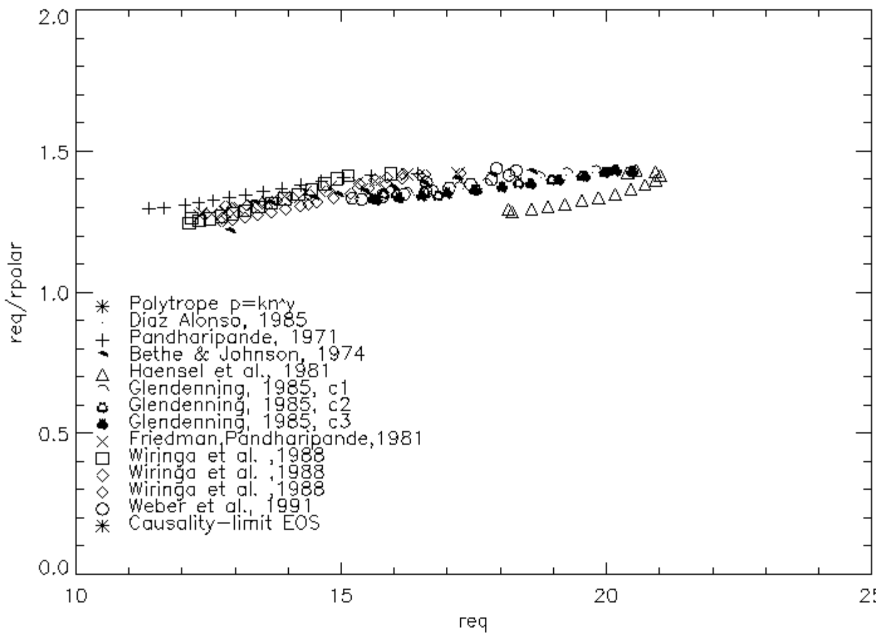


Figure 14. Comparison of equatorial radius and polar to equatorial radii ratio based upon equation (4)

The linear regression model demonstrates the variability of the intermediate values, which have the lowest coefficient of determination, slope, and significance out of the three equations (Table 5).

Table 5. Correlation (*r*), coefficient of determination ($r^2$), and slope (*b*) and significance of the linear regression line (*sig*) for left compared to right sides of intermediate rotation equation (6), maximum rotation values (3) compared to Salgado II et al.'s values, and ratio of polar to equatorial radius (4).

| Equation | *r* | $r^2$ | *b* | *sig.* |
|---|---|---|---|---|
| Intermediate (6) | 0.941 | 0.885 | 0.731 | 0.090** |
| Maximum (3) | 0.995 | 0.990 | 0.987 | 0.001* |
| Equatorial/Polar Radius (4) | 0.995 | 0.990 | 1.502 | 0.001* |

*p-value <0.01, **value approaching significance

### *4.6 Equation of State Breakdown*

Table 6 includes fourteen EOS, each of which represents a large portion of the dataset and is shared by multiple studies. Percent error pertaining to the ratio of left to right sides of equation (6) is grouped by EOS. The errors are

Table 6. Percent error for data sorted by respective EOS used by previous studies.

| Equation of State | % error |
|---|---|
| AV+14UV, Wiringa et al., 1988 | 36.303 |
| Bethe & Johnson, 1974 | 35.663 |
| Causality-Limit EOS | 45.693 |
| Diaz Alonzo, 1985 | 36.640 |
| Friedman, Pandharipande, 1981 | 39.127 |
| Glendenning, 1985: c1, c2, c3 | 34.278 |
| Haensel et al., 1981 | 38.451 |
| Pandharipande, 1971 | 29.244 |
| Pandharipande et al., 1975a, b | 32.326 |
| Polytropic Analytical EOS | 23.702 |
| UV+TNI, Wiringa et al., 1988 | 39.902 |
| Lorenz et al., 1993 | 18.182 |
| UV+UV Wiringa et al., 1988 | 46.540 |
| Weber et al., 1991 | 35.584 |

relatively large, ranging from 29.244% (Pandharipande et al., 1975a, b) to 46.540%, (UV+UV Wiringa et al., 1988), excluding the Polytropic Analytical and Lorenz et al. EOS, with values 23.702% and 18.182%, respectively.

### *V. Discussion and Conclusion*

Previous research has accurately modeled the shapes of individual pulsars (Cook et al, 1994, Friedman et al., 1986, Salgado et al., 1994), but the process by which this is done requires many complex components and steps. In addition, various EOS and numerical codes make very different predictions about neutron star shape. For this reason, more basic analytical equations are necessary to standardize the modeling of neutron star shape. Basic maximum and intermediate orbital velocity equations are relatively accurate in determining neutron star radius, though the smaller percentages for maximally rotating neutron stars demonstrate that the maximum velocity formula has a consistent, more easily corrected error than that of the intermediate cases. It is interesting to note that when divided by EOS, the error of the intermediate cases, most of which are from Salgado II et al., remains relatively high (Table 6). The smaller margin of error for the polytropic EOS could be because these data were unrealistic models. This data supports the need for further refinement of the intermediate cases.

The limitations of this research are inherent to theoretical astrophysics, which requires that assumptions be made in order to test theories. The Newtonian Roche Model for Rotating Stars assumes that the mass of a neutron star is concentrated in its center. In addition, when calculating polar radius values from equatorial radius values obtained from Salgado II et al., 1994, we assumed that the stationary neutron star's equatorial radius was equal to its polar radius, and the polar radius of a spinning body was then said to be unchanging. The complexity of our universe is also, of course, a factor; it may in fact be necessary to take into account complex factors such as general relativity through the use of separate equations that are specific to certain aspects of neutron stars. However, our promising results as well as previous research point to the success of a more uniform equation addressing many of these aspects. By simplifying the determination of neutron star shape, the study of pulse profiles of neutron stars can in turn be made easier. As neutron stars that emit X-rays rotate, the beams of radiation move in and out of Earth's view and thus vary in intensity. Understanding the shapes of these neutron stars is an important component of models of these and other observed variations.

This study can not only contribute to a deeper understanding of neutron stars, but can potentially provide valuable insight into laws of gravitation, which are still poorly understood and can be tested using pulsars. The developed equations may also be useful in the analysis of celestial bodies other than neutron stars, and a simpler equation will allow many more objects to be analyzed within a given period of time. Future research should obtain codes to test these equations using results from a wider range of models, and make the equations more accurate through further algebraic manipulation. Attempts will be made to assess other basic equations for their accuracy, such as density and volume equations, and ways in which to improve these basic equations will be explored.

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