Application of Semi-major Axis Length Analysis to the Determination of Temperature and Surface Composition of Solar System Objects in Various Stages of Solar Evolution

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ABSTRACT

Expansion of the Sun in the future will cause conditions to be vastly different from those today, leaving the Earth unsustainable. This study intends to establish which Solar System object will be most conducive to the survival of humans during five stages of solar evolution: (1) further along the main sequence at age 8.40 billion years (Ga), (2) during the red giant stage at 11.93 Ga (3) 6 million years (Ma) later prior to the helium flash (4) after the helium flash at age 12.17 Ga and (5) the beginning of core crystallization at age 12.23 Ga. The Evolve ZAMS code (Paxton, 2004) determines the mass and luminosity of the Sun at these stages. Semi-major axis lengths of each of the solar system objects (SSOs) are calculated based on mass loss (Schröder and Smith, 2008) and the principle of conservation of angular momentum. The potentially sustainable objects' temperatures are solved for using blackbody equations, from which comparison of RMS gas speed with escape velocity determines the ability of a body to retain an atmosphere consisting of a specific gas. It is found that Earth and Mars are optimal SSOs in stage 1. In stages 2 and 3, Triton is most sustainable, but in stage 4, the Galilean moons and Titan appear to be more habitable. Stage 5 has Triton being the most optimal. It is clear that more research must be conducted on these objects today to ensure the long term survival of humanity.

PERSONAL SECTION:

I've been very interested in the big questions of our existence ever since reading Stephen Hawking's a *Brief History of Time* back in seventh grade. I can't say I understood much of his ideas back then, but his clear way of conveying cosmology and our origins and ultimate fate drove me to learn more about this fascinating subject. Problems like determining the fate of the universe and humankind really garnered my interest and sitting on a beach one afternoon in Southern California watching the sunset two years ago, I began to think about our place in the Solar System and the effects on the Earth of an expanding, red giant Sun. Moreover, I realized that if the Earth becomes inhabitable due to increased solar output, the same increase in output might potentially make other objects in the Solar System (eg. Jupiter's moons or Mars) sustainable for human life. I realized that this project would require solar modeling software that would give me solar parameters (luminosity, radius, etc.) as a function of age and enable me to predict the effects of the change in these parameters on surface conditions on other objects in the Solar System.

I performed this research at home on my trusty laptop, but utilized help from the developer of the solar modeling package (Dr. Bill Paxton, UCSB) in setting up the program to create the outputs I needed. As my project dealt with solving multiple physical equations for specific values (eg. surface blackbody temperature from the Stephan-Boltzmann equation), I did not have to learn additional mathematics to complete my project, but apply relatively straightforward mathematical concepts from past years to complex physical systems. Completing this project helped me realize the vast role that mathematics plays in science, especially in the field of astronomy. It made mathematics much more engaging for me, now that I see practical application in virtually every chapter of mathematics that I learn. The best advice that I can give to those wanting to undertake a project combining math and science is to tinker around with equations, taking a blank sheet of paper and a pencil, and spending an hour figuring out the role of mathematics in your project. Whether this means solving complex differential equations or setting two algebraic equations equal to each other and solving, it can help your project become organized and mathematically sound. Also, not giving up if the math becomes too complicated is important. It's just like any other science; spending time learning concepts is necessary to make progress in your project.

1. INTRODUCTION

1.1 Rationale and Purpose

From the very beginning of civilization, humans have pondered their future through innumerable myths and legends. Through the times of the ancient Greeks' tales, with their stories of oracles prophesying the ruin of empires, and the Middle Ages, with seers like Nostradamus appearing to peek into the future, the idea of an ultimate destiny has become an obsession for many. Only nowadays do we have the scientific tools coupled with ultra-fast processing power necessary to make a well-substantiated picture of the distant future of the Solar System, which seems harbor large uncertainties for this planet. With the greatly increased radiation of an evolving Sun, our survival comes into question.

Ever since zero-age main sequence (ZAMS), the Sun has been gradually expanding to present values (Schröder & Smith, 2008). The increased luminosity from this expansion will make the Solar System's situation markedly different from today's conditions. Eventually, core hydrogen will be exhausted and outward radiation pressure will be unable to prevent gravitational collapse. The star will then contract until core temperatures reach the point where the electrostatic repulsion of helium nuclei can be overcome by particle velocities. The triple-alpha process will then begin in the core; the energy output from this will then suffice to prevent collapse, and hydrostatic equilibrium will once again commence. Just following the helium flash, the Sun will be noticeably smaller in radius and luminosity, as compared to the pre-flash state. From this point, the star will continue to expand, at a faster rate than before, reaching extremely high core densities and much higher temperatures (Paxton, 2004). Even the gradual increase in solar energy output in the early stages of late evolution will cause runaway greenhouse conditions on this planet, leading to the boiling away of the oceans and the effective destruction of the biosphere (Schröder & Smith, 2008). In several billion years, life will no longer be feasible on the planet we reside on today and thus it is of relevance to the future of living species to consider our options for survival when the Sun starts to evolve as a late-main sequence star. As this paper demonstrates, our continued existence will be contingent upon successful transfer to a more habitable environment.

1.2 Previous Work and Literature

This paper builds on an unpublished study by the author of this report. An updated and modified version of Eggleton's STARS code (descriptions can be found in Eggleton, 1971, Eggleton, 1972, and Eggleton, 1973), Evolve ZAMS developed by Paxton (2004), was previously used previously to determine solar luminosities at various stages of evolution. From these values, models were made to simulate temperature changes taking place on various Solar System Objects (SSOs) as a function of evolutionary stage. These calculated temperatures were then used to determine the existence or sustainability of several gases (H_2O, CH_4, N_2, O_2, and CO_2) on the objects' surfaces. The most troubling issue with the previous study conducted was the ignoring of mass loss in the solar model, which led to the prediction of smaller semi-major axis lengths, and therefore artificially higher temperatures. The solar wind causes the loss of approximately $3 * 10^{-14}$ solar masses per year (Lim & White, 1996) and the Evolve ZAMS program predicts an increasing mass loss rate as a function of time, as a result of lesser gravitational force holding the outermost layers of the sun's outer atmosphere (photosphere) in. Following from the principle of conservation of angular momentum (see section 2.2), these objects will migrate outward. The idea of mass loss influencing orbit distances was elaborated by the 2008 Schröder & Smith paper, primarily concerning the impact of tidal friction and angular momentum conservation on the evolution of Earth's orbit. This distancing from the Sun has the potential to leave once-discounted objects sustainable, and render previously-judged-as-optimal objects, such as a moon of Saturn, Enceladus, as too cold to maintain conditions necessary for life. Thus by providing an improved picture of the effects of solar evolution on conditions on SSOs, this paper attempts to understand the question of long term survival more accurately.

2. METHODOLOGY

2.1 Selection of solar stages, planets, gases, and operationalization

To quantify which stages of solar evolution would be studied, Paxton's Evolve ZAMS program (2004) is run with appropriate initial solar parameters of one solar mass and metallicity = 0.02. Functioning as a control in this study is model 65, representative of the Sun today. The next stage is model 100, approximately four billion years in the future when the Sun is beginning to proceed up the red giant pathway with small increases in luminosity and radius, and a slight decrease in mass. After this, model 480 is analyzed, having a nearly 60-fold increase in radius, a more than 400-fold increase in luminosity, and more mass loss. This is followed by models 521 and 523, just before and just after the helium flash respectively, both having lost substantially more mass than model 480. Model 521 has a much greater luminosity, at over 2500 times the present value, and a 182 fold increase in radius from today's. Model 523 shows a marked decrease in mass and luminosity following the helium flash, as a result of collapse in the outermost layers of the Sun triggering the triple alpha process in the core. Finally, the report concludes with model 956 with the core rapidly crystallizing, the point when the evolutionary code encounters a break. At this point, the Sun is a tip-RGB star entering the asymptotic giant branch stage, with large values of luminosity (~2800 times present) and radius (~100 times present). The Sun will have lost almost 50% of its original mass; thus the effect on planetary orbits will be significant. These models are a representative of different stages in solar evolution and provide this study with quantitative data to make calculations with (table 1).

Model	**logAge**	**logR**	**mass (m□)**	**logL**	**age(y)**	**R (R□)**	**R(m)**	**mass (kg)**	**L (L□)**	**L (W)**	**notes**
65	9.6594	0.0022	0.9992	0.0047	4.565E+09	1.005	6.995E+08	1.987E+30	1.0109	3.886E+26	current
100	9.9242	0.0791	0.9981	0.1662	8.398E+09	1.200	8.350E+08	1.985E+30	1.4662	5.636E+26	energy increase
480	10.0766	1.8217	0.9401	2.8247	1.193E+10	66.328	4.616E+10	1.870E+30	667.8824	2.567E+29	radius increase
521	10.0768	2.2624	0.7525	3.4127	1.193E+10	182.978	1.273E+11	1.497E+30	2586.4257	9.942E+29	before helium flsh.
523	10.0854	1.2807	0.7524	2.0171	1.217E+10	19.085	1.328E+10	1.496E+30	104.016	3.998E+28	after helium flsh.
956	10.0905	2.0084	0.5661	3.4469	1.232E+10	101.953	7.096E+10	1.126E+30	2798.3369	1.076E+30	crystallizing core

Table 1: Output of Evolve ZAMS for selected models. Age is the number of years from zero-age main sequence stage, R is the Sun's radius, L is the Sun's luminosity, and notes indicate the significance of the choice of each stage.

In the last several decades, our understanding of the Solar System has increased tremendously. Spectroscopic analyses of reflected light have yielded a wealth of information about the compositions of various SSOs. Such surveys have discovered that Titan's surface has cryovolcanic emissions of water and various organic compounds (Coradini et al., 2009). In the timescales discussed in this report, with the increased temperatures caused when the Sun becomes a red giant, there exists a possibility that this satellite would become capable of sustaining life as we know it. Furthermore, the Cassini mission witnessed water geysers on Enceladus, another moon of Saturn. Although the substance on this distant moon is far below its freezing point, geological processes do periodically thaw out regions of the vast sheets of surface ice present (Coradini et al., 2009). The Galilean moons, with their mass, are very likely to retain atmospheric gases and act as refuges in the red giant stage of the Sun. Additional massive satellites, like Uranus's five largest moons (Titania, Oberon, Ariel, Umbriel, and Miranda) and Neptune's Triton are considered as well for the possibility that they might be able to sustain an artificially generated atmosphere, an idea that may not seem so outlandish given the timescales involved. Also under study is Ceres, a dwarf planet in the asteroid belt and Mars, two SSOs with conditions that the Sun's changes would almost certainly affect. An attempt is made to select objects from representative sections of the Solar System, (eg. inner planetary, inter-asteroid belt, Jovian, Saturnian, Uranian, Neptunian regions) as the specific effects of the Sun's luminosity fluctuations over time on objects at various distances is itself unclear. The gases studied (H_2O, CH_4, N_2, O_2, and CO_2) are selected for their relevance to life and abundance on SSOs.

It is reasonable to define "habitable" in this context as "being capable of supporting human life". To quantify this, complex biological forms have several basic necessities: water, oxygen, and a stable climate. Average temperatures of between 230K and 320K are required for survival for most species (Rothschild & Mancinelli, 2001). Although the extremes of this range are generally considered too harsh for human survival, increased technological capabilities of the future have the potential to make the entire range conducive to human life.

2.2 Semi-major axis length calculations

The total angular momentum of an SSO involves two main sources of circular motion: (1) rotation around its own axis and (2) revolution around the Sun. A combination of the comparatively much larger angular momentum produced by the latter revolution and importantly, the braking of rotation caused by atmospheric drag and tidal effects makes the rotational component insignificant in large timescales (Lewis 1994). Thus, this paper will concern itself with the effect of the bleeding away of solar mass on conservation of revolution-based momentum.

As the Schröder & Smith paper (2008) explains, the semi-major axis length of an object (effectively its radius) is given by the following equation:

$$r_{obj} = \frac{{L_{obj}}^2}{{M_{obj}}^2 \mathbf{G} * \mathbf{M}_{sun}}$$

(1)

The semi-major axis length is given by r_{obj} , L_{obj} represents the angular momentum of the object, M_{obj} is the object's mass (table 2), $\mathbf{G}$ is the Newtonian gravitational constant (6.67428 x 10^{-11} m^3 kg^{-1} s^{-2}), and $\mathbf{M}_{sun}$ is the mass of the Sun corresponding to the particular stage in question. Given that L_{obj} is conserved (as per the principle of conservation of angular momentum) as the Sun evolves, $\mathbf{M}_{sun}$ becomes the only variable.

Revolution-based angular momentum L is determined using the equation $L = r \times mv$ for all objects revolving directly around the Sun (table 2). This is then used as the L_{obj} term in equation 1. Using Evolve ZAMS's outputs for $\mathbf{M}_{sun}$ (table 1), r_{obj} is determined for each considered stage of solar evolution and semi-major axis lengths are calculated for each object in each stage of evolution considered.

Object	**mass (kg)**	**orbit time (y)**	**orbit time (s)**	**semi-major axis (m)**	**orbit circumference (m)**	**orbit velocity**	**L = mvr**
Earth	5.972E+24	1.000	3.156E+07	1.496E+11	9.39951E+11	29785.3757	2.66111E+40
Mars	6.417E+23	1.881	5.935E+07	2.279E+11	1.43218E+12	24129.61535	3.52936E+39
Ceres	9.430E+20	4.600	1.452E+08	4.138E+11	2.60019E+12	17912.32799	6.99018E+36
Jupiter	1.898E+27	11.863	3.743E+08	7.785E+11	4.89176E+12	13067.41514	1.93108E+43
Saturn	5.683E+26	29.447	9.293E+08	1.433E+12	9.00663E+12	9692.127844	7.89576E+42
Uranus	8.681E+25	84.017	2.651E+09	2.877E+12	1.80747E+13	6817.26566	1.70244E+42
Neptune	1.024E+26	164.791	5.200E+09	4.503E+12	2.8296E+13	5441.2111	2.50947E+42

Table 2: Table of angular momentum calculations, L_{obj}, for each object revolving directly around the Sun

The Schröder and Smith study (2008) also expounds on the effects of torque (produced by retarded photospheric tidal bulges) on the total angular momentum of a Sun-object system. Net torque is given by an equation (Zahn, 1989, eq.11) with the form $\tau = n\left(\frac{R_{sun}}{r_{obj}}\right)^{6}$ where n is a constant, R_{sun} is the radius of the Sun, and r_{obj} is the object's semi-major axis length. When $r_{obj} \gg R_{sun}$ as for all objects beyond 1 AU, τ becomes insignificantly small and has little effect on orbit decay. Since this paper focuses on the search for a sustainable object, and Mars and Earth are never likely to become inhabitable in the RGB stage of the Sun's evolution, torque caused by tidal bulges is ignored in semi-major axis length determinations.

2.3 Blackbody temperature calculations

From the lengths of the objects' semi-major axes predicted in 2.2, it is possible to approximate surface temperatures on the SSOs using the following equations and relations (Zeilik, Gregory, & Smith, 1992):

$$E_{incident} = L_{sun} * \frac{\pi r^2}{4\pi R^2} \qquad (2)$$

Eq. 2 describes the incident solar luminosity on an object with radius r and semi-major axis R. By visualizing each solar system object as an object on the surface of an imaginary sphere with the Sun at its center with surface area $4\pi R^2$ and observing that by conservation of energy, the energy received by the surface of the entire sphere is equal to solar luminosity, eq. 2 shows incident radiation as a fraction of total solar luminosity, L_{sun}.

$$E_{absorbed} = (1 - \alpha) * L_{sun} \frac{\pi r^2}{4\pi R^2} \qquad (3)$$

Eq. 3 is the product of eq. 2 with the object's geometric albedo and the total energy absorbed by the object is calculated.

$$j = \sigma T^4 \qquad (4)$$

Eq. 4 is the Stefan-Boltzmann law stating that a blackbody's irradiance j (per square unit) equals a constant σ (5.670400 x 10^{-8} W m^{-2} K^{-4}) multiplied by the fourth power of the object's temperature T.

$$E_{output} = 4\pi r^2 * \sigma T^4 \qquad (5)$$

By multiplying eq. 4 by the total surface area of the planet, eq. 5 represents the energy output by a blackbody of radius r per unit time.

$$(1 - \alpha) * L_{sun} \frac{\pi r^2}{4\pi R^2} = 4\pi r^2 * \sigma T^4 \qquad (6)$$

Setting eqs. 3 and 5 together, thermal equilibrium is simulated, with total energy absorbed being equal to the total energy radiated.

$$T = \left(\frac{(1-\alpha) * L_{sun}}{16\pi\sigma R^2}\right)^{\frac{1}{4}}$$

(7)

Cancelling like terms and solving for object temperature, eq. 7 is derived. This final formulation of thermal equilibrium analysis is used to approximate the surface temperatures on all SSOs considered in the study of all relevant stages of solar evolution. The relevant data used to derive the surface temperature are given in table 1 (solar luminosity), table 3 (albedo), and table 5 (semi-major axis length).

Object	Earth	Mars	Ceres	Io	Europa	Ganymede	Callisto	Titan	Enceladus	Miranda	Ariel	Umbriel	Titania	Oberon	Triton
Albedo	0.37	0.16	0.073	0.63	0.67	0.43	0.17	0.20	0.99	0.32	0.39	0.21	0.27	0.23	0.76

Table 3: Geometric albedo values of each of the SSOs considered (JPL)

2.4 Gas retention analysis

From the temperatures determined by section 2.3 of this report, it is possible to determine the possibility of sustenance of an atmosphere on SSOs through solar evolution. By statistical methods (Seligman 2001) it is calculated that in order for a gas to be retained on an object for over 10,000 years, the body's escape velocity must be at least three times the gas's root mean square speed. This comes as a result of the definition of temperature: the average kinetic energy of gas molecules. As this is an average, there is a significant fraction of gas molecules with greater speed than RMS, and thus are able to overcome gravitational attraction and escape into space.

$$V_{esc} = \sqrt{\frac{2GM_{obj}}{r}} \tag{8}$$

Eq. 8 is the formula for the escape velocity of an object, where G is the Newtonian gravitational constant (6.67428 x 10^{-11} m^3 kg^{-1} s^{-2}), M_{obj} is the mass of the SSO under consideration, and r is the object's radius.

$$V_{1000_{0}y} = 3 * \sqrt{\frac{3\mathbf{R}T_{max}}{M_M}} \tag{9}$$

Eq. 9 represents three times the root-mean-squared speed of a gas with molar mass M_M at maximum temperature T_{max}. $\mathbf{R}$ is the molar gas constant (8.314472 J mol^{-1} K^{-1}). This speed is the maximum RMS speed at which a gas can exist at on an object without significant quantities escaping in 10,000 years (Seligman).

$$T_{max} = \frac{2GM_{obj}M_M}{27\mathbf{R}r} \tag{10}$$

To calculate the gas's maximum retention RMS speed, eqs. 8 and 9 are set equal to each other and squared. T_{max} is solved for to derive eq. 10, representing the highest temperature possible for a particular gas to be retained on the SSOs surface for 10,000 years. This is applied to each gas considered (H_2O, CH_4, N_2, O_2, and CO_2) on each object and then compared to the surface temperature calculated in 2.3 to determine sustainability of an atmosphere of a specific composition. The relevant characteristics in this calculation are given in tables 2 (M_{obj} and r) and 4 (M_M).

gas	**CH_4**	**H_2O**	**N_2**	**O_2**	**CO_2**
molar mass	16.05	18.02	28.02	32.00	44.01

Table 4: Molar masses of gases used in section 2.4.

3. RESULTS

3.1 Semi-major (radius) axis length calculations

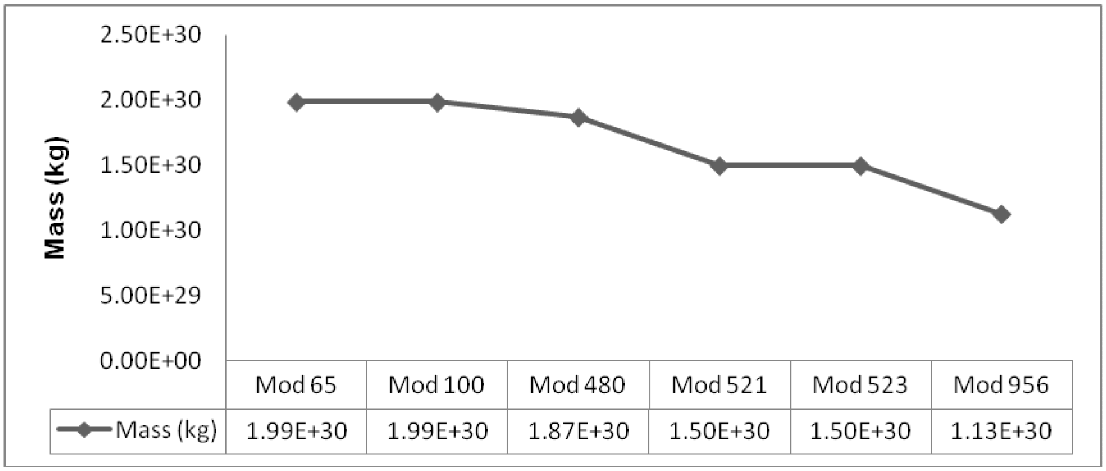

Figure 1: Mass of the sun as a function of model number. These values are calculated by the Evolve ZAMS program (Paxton, 2004).

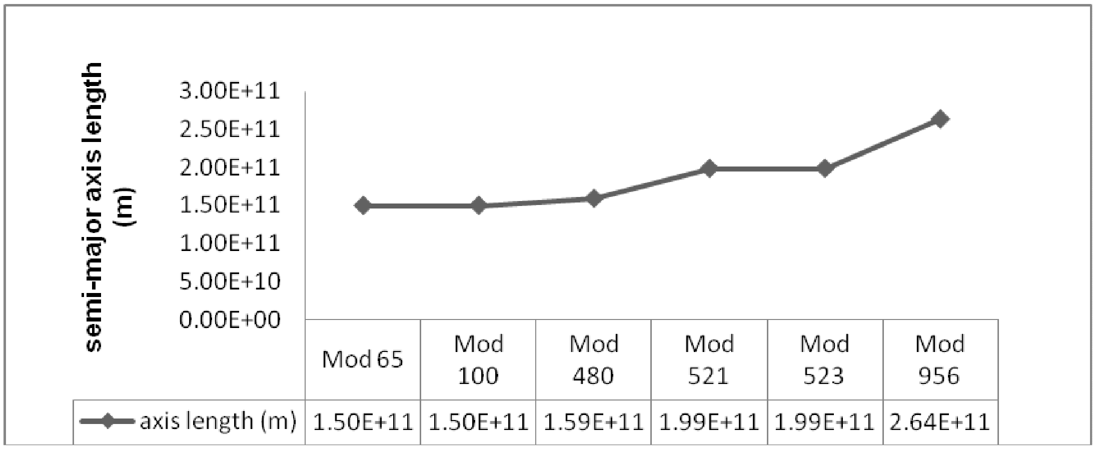

Figure 2: Earth's semi-major axis length as a function of model number

Object	**Mod #65**	**Mod #100**	**Mod #480**	**Mod #521**	**Mod #523**	**Mod #956**
Mars	2.28E+11	2.28E+11	2.42E+11	3.03E+11	3.03E+11	4.03E+11
Ceres	4.14E+11	4.15E+11	4.40E+11	5.50E+11	5.50E+11	7.31E+11
Jupiter	7.80E+11	7.81E+11	8.29E+11	1.04E+12	1.04E+12	1.38E+12
Saturn	1.46E+12	1.46E+12	1.55E+12	1.93E+12	1.93E+12	2.57E+12
Uranus	2.90E+12	2.90E+12	3.08E+12	3.85E+12	3.85E+12	5.12E+12
Neptune	4.53E+12	4.53E+12	4.81E+12	6.01E+12	6.01E+12	7.99E+12

Table 5: As each graph has the same trend of increasing semi-major axis, it would be an exercise in futility to graph this for each object. This table shows the objects' average distance from Sun (semi-major axis length) as a function of stage of stellar evolution.

3.2 Blackbody temperature calculations

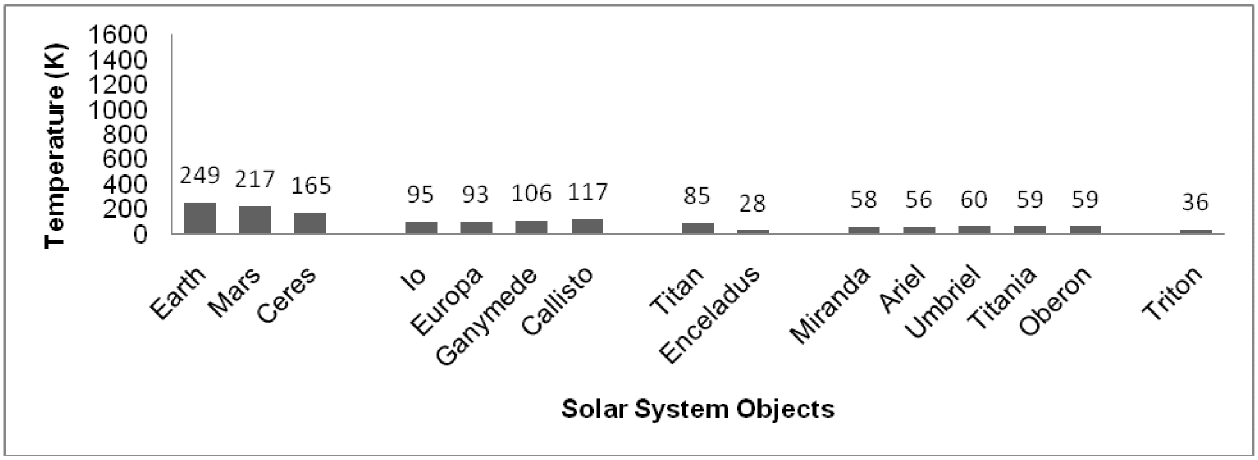

Figure 3a: Temperatures of SSOs in model 65

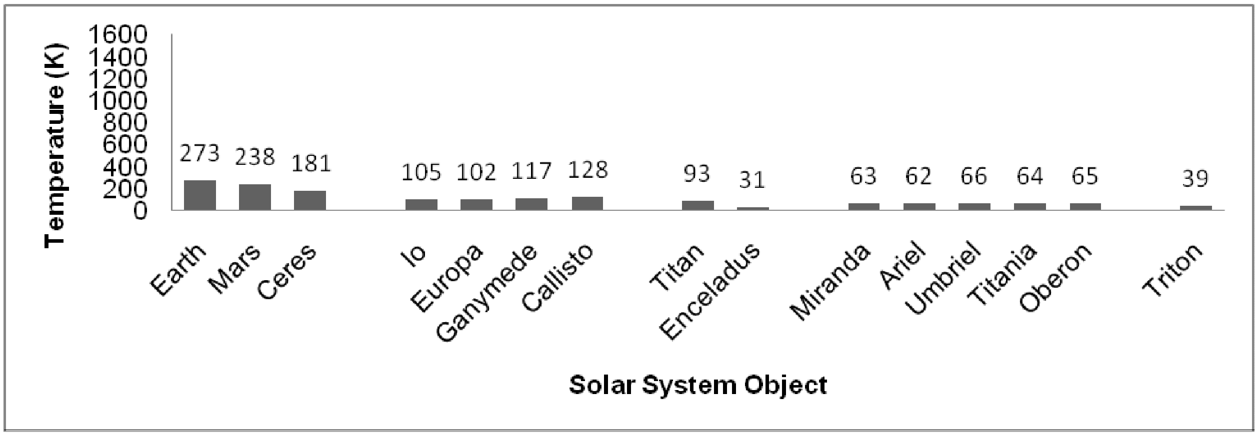

Figure 3b: Temperatures of SSOs in model 100

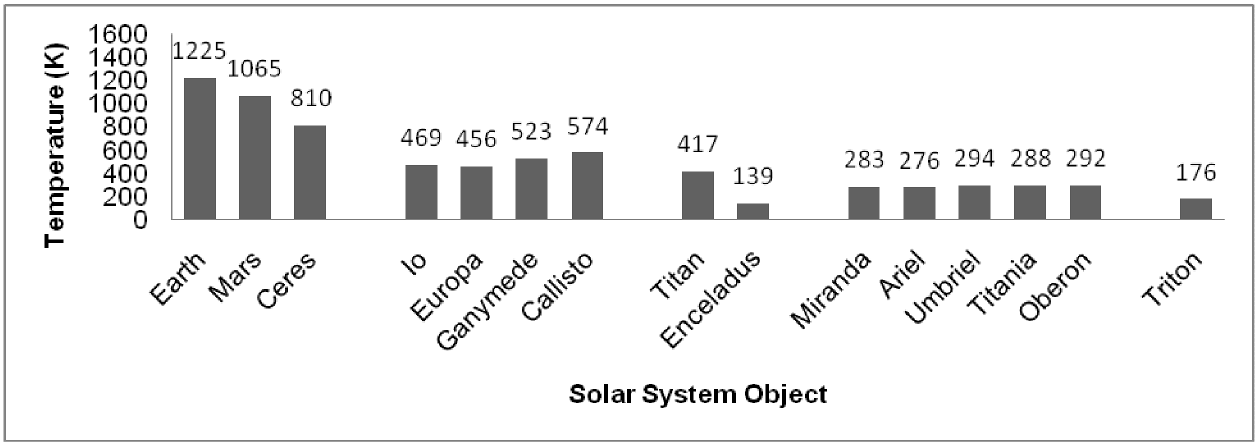

Figure 3c: Temperatures of SSOs in model 480

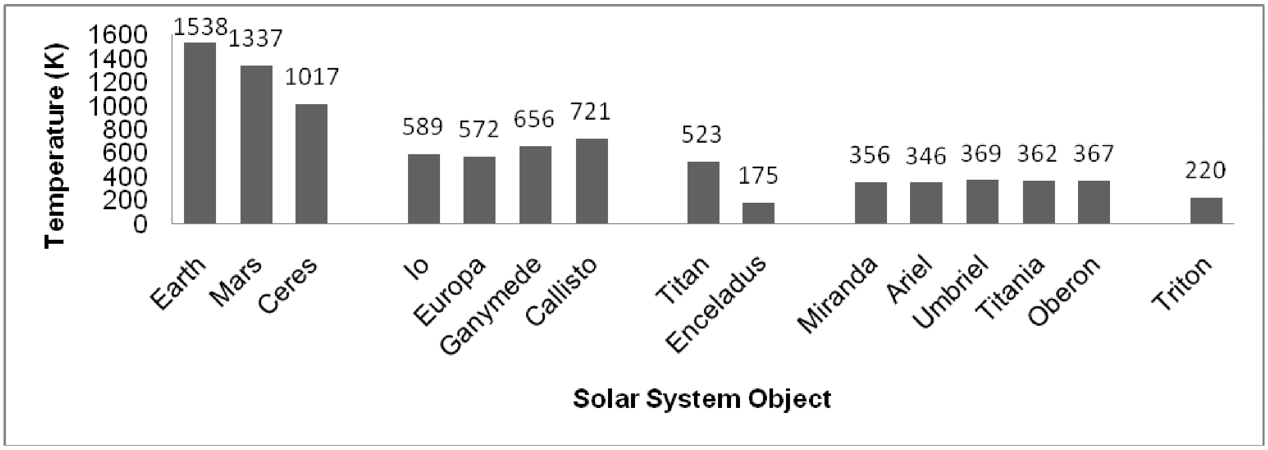

Figure 3d: Temperatures of SSOs in model 521

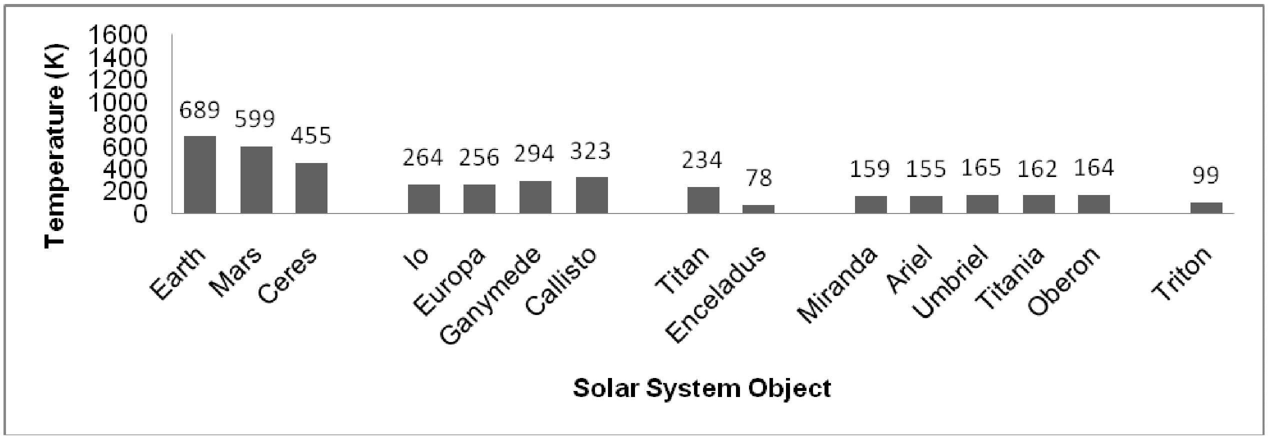

Figure 3e: Temperatures of SSOs in model 523

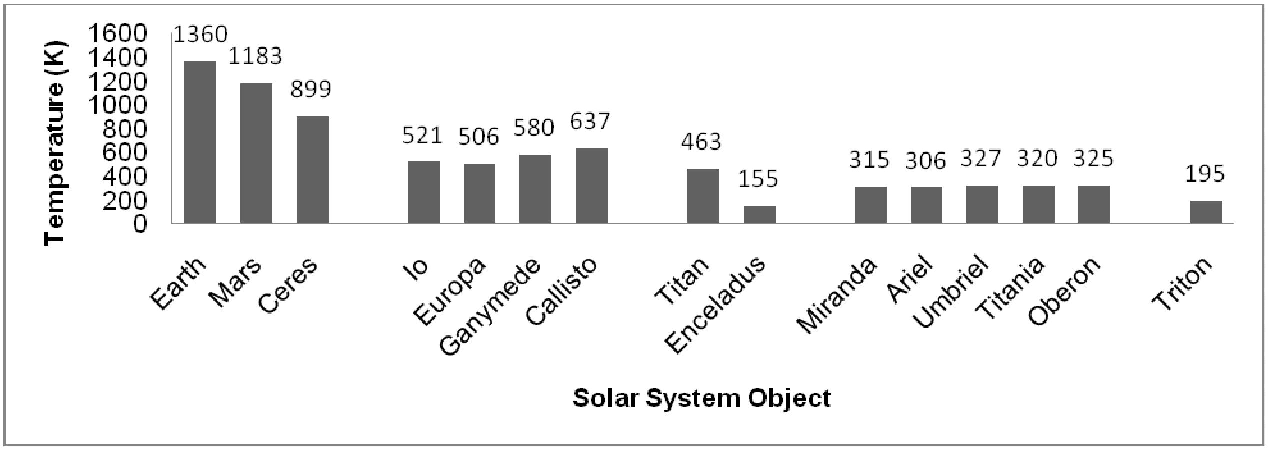

Figure 3f: Temperatures of SSOs in model 956

3.3 Results of gas retention analysis

Object	CH_4	H_2O	N_2	O_2	CO_2
Earth	8938.1	10041.4	15608.2	17831.6	24524.0
Mars	1804.9	2027.7	3151.8	3600.8	4952.2
Ceres	18.4	20.7	32.2	36.8	50.6
Io	469.1	527.0	819.1	935.8	1287.1
Europa	291.6	327.6	509.2	581.8	800.1
Ganymede	536.3	602.5	936.5	1069.9	1471.4
Callisto	426.8	479.5	745.3	851.5	1171.0
Titan	498.0	559.4	869.6	993.4	1366.3
Enceladus	4.1	4.6	7.2	8.2	11.3
Miranda	2.7	3.0	4.7	5.4	7.4
Ariel	22.2	25.0	38.8	44.3	61.0
Umbriel	19.6	22.0	34.2	39.1	53.7
Titania	42.6	47.9	74.4	85.0	116.9
Oberon	37.7	42.4	65.9	75.3	103.6
Triton	150.8	169.4	263.3	300.8	413.7

Table 6: By the method discussed in section 2.4, these are maximum temperatures necessary for retention of specific gases for each SSO for 100 million years.

4. DISCUSSION

4.1 Temperature

As mentioned in the operationalization section of the introduction, a "livable" temperature is defined as a blackbody approximation temperature of between 230 K and 320 K (Rothschild & Mancinelli, 2001). The calculations (see 2.2 and 2.3) done to produce fig. 3.1 determine Earth's present blackbody temperature to be 249 K, which is 38 K less than the average surface temperature of 289 K on this planet today. This is a result of greenhouses gases trapping additional heat (Stewart, 2005). At the much larger time scales accounted for in this study, 38 K is not of much significance. In addition, the lack of a climate model predicting precise atmospheric conditions in the coming few billion years means that models made in this study must assume perfect blackbody conditions.

Refer to table 1 for exact predicted solar values at the model numbers studied. At the time of model 100, approximately four billion years from present, the relatively minor growth in solar radius and correspondingly increased luminosity would make Mars, in addition to Earth, the only two objects considered to be livable in terms of temperature (between 230K to 320K) at a blackbody temperature of 238K (see figure 3b). Around 3.5 billion years later, the growing Sun will render the inner Solar System completely unlivable, with even the Galilean moons and Titan being too warm for sustenance. The Uranian satellites (see fig. 3c) will however have temperatures in the range of 230 K - 320 K. In only six million years, the Sun will triple its radius as it continues to grow as a red giant star in model 521 (fig. 3d). Here, no object can maintain temperatures in the range, with the best possibilities being the Uranian moons (too hot), and Triton (too cold). Immediately following the helium flash (model 523), the Sun's radius will shrink dramatically, and although the solar surface will be hotter, the overall luminosity will be markedly lower (Paxton, 2004). Here, all four large moons of Jupiter and Saturn's Titan maintain temperatures deemed necessary for the sustainability of life. As the Sun's core begins to crystallize in model 956, the Sun's radius once again grows with a much greater luminosity as a result of both greater radiative surface area and inherently greater flux. Although the radius is less than that of prior to the helium flash (model 521), the greater flux causes a higher luminosity. However the mass loss (fig. 2) will lead to a lengthening of semi-major axes (fig. 2 and table 5), causing overall temperatures on the SSOs to be substantially lower in model 956 in comparison to model 521. The Uranian satellites discussed are the still best options in this period in terms of temperature, albeit lower in the temperature range.

4.2 Gas Retention

The idea of "terraforming" an object to simulate Earth-like conditions will probably be reality in the timeframes discussed in this study (Boston, Ivanov, & McKay, 1992). Terraforming involves the placement of extremophilic organisms in unfavorable environments to create atmospheres and surface chemicals necessary for human survival. Based on this, this report does not intend to answer the question of whether these SSOs will *have* the required composition but rather, whether gases can be *retained* if produced.

The large planets considered, Earth and Mars, have sufficient mass to maintain an atmosphere at all stages of solar evolution (fig. 3 graphs and table 6). Ceres, owing to its very low mass cannot hold these gases at any stage. In the Galilean moon system, all objects can retain these gases today and in model 100, while only Io and Ganymede can provide sufficient gravitational force and low enough temperature to maintain all gases in model 480. In model 521, no Galilean moon can retain CH_4 or H_2O. Europa cannot retain N_2, but can retain O_2 and CO_2. Io, Ganymede, and Callisto can retain N_2, O_2 and CO_2. Post-helium flash (mod. 523), all gases can be retained on the Galilean moons. In stage 956, with the Sun as a near-tip RGB star, all mentioned gases but CH_4 can be retained Jupiter's four largest moons, with the exception of H_2O on Callisto. Moving on to the Saturnian satellites, Enceladus does not have sufficient mass to ever maintain an atmosphere. With the exception of CH_4 in stage 521 when Titan has its hottest temperatures, Titan can retain all gases discussed in all relevant stages of solar evolution. With the Uranian satellites, Miranda, Ariel and Umbriel cannot retain (with the exception of CO_2 on Ariel today) any gas at any stage. Titania and Oberon can retain atmospheres of N_2, O_2, or CO_2 for stages 65 (today) and 100, but cannot retain these gases in the stages after. More optimistically, Triton can maintain all gases now, at stage 100, and also at stage 523, and only N_2, O_2, or CO_2 for the remaining stages 480, 521, and 956.

4.3 Overall Discussion

Although Earth and Mars maintain an atmosphere in all stages of solar evolution, the greatly increased temperatures make these SSOs unsustainable for our future in the Solar System. Ceres, by being both too warm and not being able to retain any of the gases discussed, can also be ruled out as a possible refuge. The four largest moons of Jupiter and Saturn's Titan can function as habitable satellites only for the post-helium flash stage of the Sun, being able to retain atmospheres and maintaining moderate temperatures. Specifically, Titan, with its preexisting atmosphere (Coradini et al., 2009), is probably our best hope in model 523, requiring the least modification to become sustainable. Enceladus, though having surface ice right now (Coradini et al., 2009), is gradually losing water through sublimation, a process that will accelerate as solar output increases. As the satellite cannot maintain any atmosphere, it will lose H_2O to space. Its low temperatures caused by extremely high albedo additionally make it a poor choice for humanity's future. Uranus's moons can never sustain an atmosphere, so although having adequate temperatures in stages 521 and 956, they ought not to be considered in future contingency planning. Triton, although deemed too cold for all of the considered timeframe, can retain an atmosphere. As discussed further in the conclusions section, the use of technology to create artificial temperatures may be necessary to heat the object to a sustainable temperature.

5. CONCLUSIONS

5.1 Future Work

This paper takes into account the effects of angular momentum conservation on the orbits of SSOs and the atmospheric and temperature-relevant consequences of varying solar radiation on the biological sustainability of these objects. It has been found that Titan, and to a lesser extent, the Galilean moons are suitable habitats in the Solar System immediately following the helium flash. Although Triton will be too cold to be sustainable, it is able to maintain an atmosphere of O_2, N_2, or CO_2, throughout all stages of solar evolution. The latter of these gases is a greenhouse gas and could perhaps increase Triton's temperature to more habitable levels. Additional modeling with atmosphere-mediated temperature increases is required and would be conducive to determining a more comprehensive plan for humanity's survival in the distant future. Similarly, the model proposed in this paper does not take into account changes in albedo that are very possible, caused by increased temperatures occurring as results of a more luminous Sun. More evaporation could increase cloud cover, thus inducing albedo changes. A decreased in absorbed radiation caused by light reflected from clouds would decrease an object's temperature, perhaps changing its ability to sustain life.

As core crystallization prevented Evolve ZAMS from modeling the Sun to past the tip-red giant stage (Paxton, 2004), a more complete model would include the Sun's late asymptotic giant branch stage. With the radius of the Sun continually growing after the helium flash, a study of an AGB Sun would most likely lead to additional objects (perhaps even Triton) being declared too hot for human life, making a search for a sustainable SSO even more important.

5.2 Applications

Even after the AGB Sun, future stages also ought to be modeled, including possibly the Sun as a white dwarf. Although the comparatively miniscule luminosity of a white dwarf would most likely render the Solar System completely unlivable, it would be instructive to quantitatively study the generalizations of this to other systems. This has applications in the search for extraterrestrial life, as the fate of civilizations surrounding white dwarf stars would be made clearer.

Furthermore, the fact that the Galilean moons, Titan, and Triton appears to be solutions to humanity's long term needs necessitates more research devoted to the moons of the gas giants. Although billions of years in the future may seem like an exceedingly long time period, it is never too late to ponder the questions of the future. Cataclysmic events before then, including cometary and asteroidal impacts may force an earlier departure from this planet, and with the additional information from such research, humanity would be far better off.

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Numerical data obtained:
2006 CODATA Values:
    http://physics.nist.gov/cgi-bin/cuu/Value?r (molar gas constant)
    http://physics.nist.gov/cgi-bin/cuu/Value?bg (gravitational constant)
Solar System Dynamics, Jet Propulsion Laboratory:
    http://ssd.jpl.nasa.gov/?sat_phys_par (parameters of satellites)
    http://ssd.jpl.nasa.gov/?planet_phys_par (parameters of planets)