Trajectory Optimization for Continuous Solar Flight

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Personal Section

From my youngest years, I have always loved math, and spent much of my elementary and middle school years with Math Olympiads and Mathcounts, which emphasized creativity and quick thinking rather than the procedural methods in school. In these years, I developed an interest in machines as well, and particularly enjoyed building model airplanes as part of a small class in our local community center. I constructed each plane out of balsa wood, and spent several weeks carefully pinning and gluing components together to make different types of planes. The most interesting thing about these planes was the simplicity of their power system, as the propellers were simply attached to tightly wound rubber bands, whose tension provided the energy needed for flight. Every so often we would have flight endurance competitions, and I was always interested in why some types of planes seemed to consistently beat others. Young as I was, I did not really have the math and physics background to understand much about the details of aeronautics, but flight would remain a subject of constant curiosity.

In high school, I gained a solid foundation in mathematics and physics, and especially learned a lot about numerical methods and differential equation systems from my father, which would prove useful in my research project. Still interested in aeronautics, I contacted a couple of professors at Stanford and was delighted when Professor Antony Jameson of the Aerospace Computing Lab invited me to attend his lab meetings, in which Ph.D. candidates would present their research. I understood little in the beginning, but eventually learned enough about the basics of flight to start my own project. After surfing the web for a while, I found a problem on maximizing the range of a hang glider subjected to a certain wind current and tried to see if I could obtain similar results. After 3 months or so, I found that my results pretty much matched up with those of the original authors, though I developed better computational methods and found some interesting new results. This investigation helped me develop, test, and refine the trajectory optimization methods I applied throughout my research project. Then, inspired by the Solar Impulse project and my childhood fascination for high endurance flight, I began the core of my work: optimizing the flight of a solar UAV to enable continuous, or theoretically perpetual, flight. I did most of my research at home on my laptop, since it was mainly computational in nature. However, I made regular trips to Stanford to receive feedback on my work from Ph.D student Manuel Lopez at the Aerospace Computing Lab.

What intrigued me most about my research was the fact that it could be easily applied to real-world problems in aviation, because as far as I know, detailed optimization of solar aircraft has thus far received little attention and existing research on the topic is inadequate. Therefore, my interest in the research did not stop once I had finished my work for the Intel competition [1]. As part of the Stanford UAV Club, I, along with the graduate students I had consulted throughout my project, began the Stanford Solar UAV Project, our goal being to construct a high endurance solar aircraft with complete design and trajectory optimization based on my research work. To do this however, I had to update my software substantially. Initially, my system determined the smallest battery that could be loaded into the airplane while finding the most energy efficient trajectory for the aircraft. However, now I also had to perform optimization to determine the ideal design and specifications of this solar aircraft, beyond just minimizing the size of the battery. For example, we optimized the wing area, total mass of the plane, aspect ratio, and the percentage of the wing area covered with solar panels. I ran a variety of cases on this new model, varying the day of year, solar panel efficiency, panel mass density, and the base mass of necessary aircraft components. Furthermore, after optimization of the design, I needed to create a system which when given the current position, would continually generate the next place for the aircraft to go to continue on the optimal trajectory.

After nearly 6 months of this work, our team has begun prototyping and planning wing designs, and we already have done significant research on the batteries and solar panels to use in the UAV. What I thought would just be a fun project to keep me occupied for a little while turned into my main focus for the past year, and has given me continuous excitement and exploration. If I were to advise other students interested in conducting research in mathematics and science, I would tell them that these types of projects are of the nature that, with enough drive, you can always find something fascinating to investigate. Thus, I would suggest that it is wise to never really have a particular end goal, or if you must have goals, simply set several short term goals, because no matter how finished you think you are, there will almost always be something left to consider, or something to improve upon. In hindsight, I feel that my Intel project is actually quite incomplete, and although my research has progressed significantly since then, there is still much more I would like to do.

Research

Introduction

In this paper, I describe the process and results of my study on the flight trajectory optimization of a *continuously* flying solar aircraft. Continuous flight is achieved by cyclic operation, where the trajectory is repeated indefinitely, typically every 24 hours. The word *continuously* is used in the theoretical sense, as continuous or perpetual flight is not achievable in practice due to degradation of batteries and aircraft components over time.

The importance of flight trajectory optimization has been recognized in both general aviation and space applications [2]. The prevalent class of algorithms for solving these problems are largely sequential in nature, where the differential equations that describe flight motion are solved in an inner loop while an outer loop performs the optimization of the control variables. These methods can be computationally expensive as they require repeated solution of the differential equations for each guess of the control variable in addition to calculation of gradients for the optimizer [3]. The algorithm may also terminate if the differential equation solver fails at intermediate guesses of the control variables given by the optimizer. Also, it is very difficult to enforce constraints on the state variables. For the optimization of solar aircraft, these sequential methods face unique challenges because the boundary conditions are not only unknown, but are required to be identical due to cyclic operation. Solving the differential equations and performing the optimization simultaneously can address these drawbacks [4, 5], but this requires good initialization of the state variables. In this research, I built upon a simultaneous solution method called orthogonal collocation on finite elements [5] to develop a robust trajectory optimization system with an effective initialization strategy.

The simultaneous method mentioned above can be applied to many flight trajectory optimization problems. The application to solar flight is uniquely interesting from an optimal control perspective since the available power is time dependent. The design of solar vehicles with batteries that can offer continuous flight has seen a lot of interest [6, 7, 8] and the recent cross continental flight of the Solar Impulse [9] has marked a major milestone in solar aviation. Here, during the day solar power is used to propel the aircraft and to store energy in batteries. The battery energy is then used for continued flight at night. Such aircraft offer unique opportunities, especially for unmanned flight, and can be used for communications, imagery, surveillance, and assistance during natural disasters. While the design of such aircraft has seen a lot of interest, there has been little reported in terms of the trajectory optimization for these aircraft. One notable attempt [10] has been made in this regard; however, the battery mass does not appear to be considered in their work, results are given for only one scenario, and computational performance results are not reported.

I addressed this gap by developing a reliable and computationally efficient system that can determine the optimal trajectory for continuous solar flight. The primary goal is to minimize the mass of the battery needed while allowing sufficient energy to make it through the night. To do this, the development of an efficient algorithm is critical. After developing and testing this algorithm, I created a detailed mathematical model for the solar aircraft. The resulting system allows a user to find the optimal trajectory for continuous flight given input parameters such as payload, battery efficiency, altitude limits, latitude, day of year, and other aircraft specifications. In addition, limiting cases have also been studied to precisely establish the latitude range within which continuous solar flight is theoretically possible.

I began my research by testing my solution algorithm on a glider range maximization problem proposed in [11], which I used as a benchmark for evaluating the method. Solving this problem allowed me to make crucial refinements to my initial trajectory optimization system, improving the accuracy and computational performance. This paper will only describe the formulation and the results of the solar aircraft trajectory optimization problem. Details of the solution method are provided in [1].

Development of Solar Aircraft Trajectory Optimization Model

The objective in this problem is to find the smallest battery capacity at which perpetual solar flight can be achieved and then chart out the optimal trajectory for the aircraft. This is done by optimizing the propeller thrust, battery charging and discharging rates, and the aircraft lift coefficient. The mathematical model is synthesized by combining the equations for atmospheric effects [12], flight dynamics [13] solar energy conversion [14], and battery operation. The atmospheric effects refer to equations relating air density with altitude, which are important since the lift and drag forces depend on air density. The flight dynamics equations are the main differential equations that govern the physics of motion. They are derived from the free body diagram of the aircraft and relate the state variables (velocity, flight angle, altitude, and horizontal displacement) to the thrust control variable. The solar energy conversion equations quantify the flow of power from the sun to the propeller and battery as shown in the schematic below.

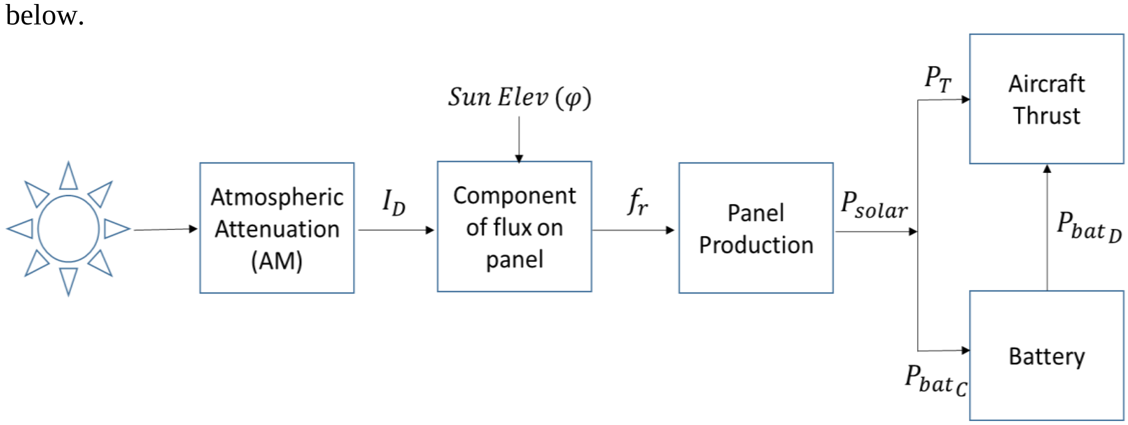

Fig. 1 Power Flow Chart

Here, power from the sun is attenuated by travel through the atmosphere, resulting in a value for the solar flux within the atmosphere (I_D). However, as a result of the elevation angle of the sun, only a portion of the solar flux (f_r) generates electrical power. Then, some of the power produced by the panel (P_{solar}) goes to the propeller (P_T) while the remaining goes to the battery (P_{bat_C}). When necessary, power is discharged from the battery to the propeller (P_{bat_D}). Finally, the battery operation equation contains the differential equation relating the battery energy state variable to the power charging and discharging rates.

In summary, the profiles of the thrust, lift coefficient, the battery charging and discharging rates are optimized and the trajectories of the state variables described above are calculated while minimizing the battery size. In this model, the hardware parameters (solar panel mass density (0.840 $\frac{kg}{m^2}$), battery energy density (1260 $\frac{kJ}{kg}$), panel efficiency (29.5 %)) are based on currently available technology. The mass of the aircraft I consider has 3 components: the airframe mass (136 kg), the solar panel mass (30.24 kg), and the battery mass (optimized).

In my optimization, I minimize the battery energy at sunset to effectively minimize the maximum battery capacity and thus the battery mass. However, the point of maximum stored energy occurs slightly before sunset. At sunset, the solar flux is zero and in the period leading to sunset, some battery energy is used up by the aircraft. Determining the exact point of this maximum storage and then minimizing this value is computationally expensive; however, this determination is not necessary. When the energy stored at sunset is minimized, so too is the maximum energy stored, thus practically achieving the same objective with vastly improved computational performance. The cycle is assumed to be a 24 hour cycle and is discretized into finite elements. The values of the control variables (thrust, battery charging and discharging rates, lift coefficient) were optimized within each finite element. In addition, to prevent sharp changes in the control profiles, the control variables were held constant within each finite element and constraints on their variation between finite elements were enforced. The entire set of equations were solved with the procedure I developed in the beginning stages of my research and implemented in the General Algebraic Modeling System (GAMS) [15].

Results and Discussion

In this section, I will illustrate the effects of key variables such as the altitude limits, day of year, latitude, panel efficiency, payload, and battery efficiency on the minimum required battery capacity, which is the objective function. Various case studies were performed to this end whose results are shown in Table 1 below. I chose a default case (**bolded** in Table 1) in order to compare against other cases. Each case modifies only the indicated parameter from the default, making it easy to establish causal links. The minimum amount of energy storage needed at sunset is reported for each case. The solution method is quite robust, as the set of diverse cases were solved with ease.

Table 1: Summary of case studies and results

Minimum required battery capacity (in kJ) is reported for each case

1. Altitude Range (m)	Min. Battery Size	2. Day of Year	Min. Battery Size
1,000 to 6,000	11470 kJ	79 (Spring Equinox)	14039 kJ
1,000 to 8,000	**7832 kJ**	172 (Summer Solstice)	7731 kJ
1,000 to 10,000	5943 kJ	**180**	**7832 kJ**
		265 (Fall Equinox)	13938 kJ
		355 (Winter Solstice)	20624 kJ
3. Latitude	**Min. Battery Size**	**4. Solar Panel Efficiency**	**Min. Battery Size**
0° (Equator)	13755 kJ	22%	8025 kJ
37° N (San Francisco)	**7832 kJ**	**29.5%**	**7832 kJ**
5. Payload (kg)	**Min. Battery Size**	**6. Battery Efficiency**	**Min. Battery Size**
0	**7832 kJ**	50%	16145 kJ
30	10761 kJ	75%	10242 kJ
50	13057 kJ	**96%**	**7832 kJ**

To understand how variables such as velocity, flight angle, thrust, battery energy, and power usage change over the duration of the 24 hour (86400 s) cycle, graphs are shown for the default case (**bolded** in Table 1) in Fig. 2 to 8.

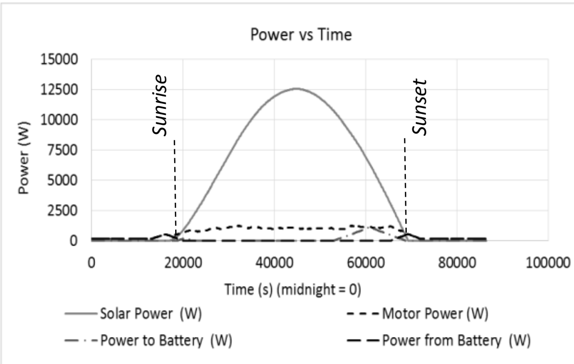

Fig. 2 Power Profiles for Default Case

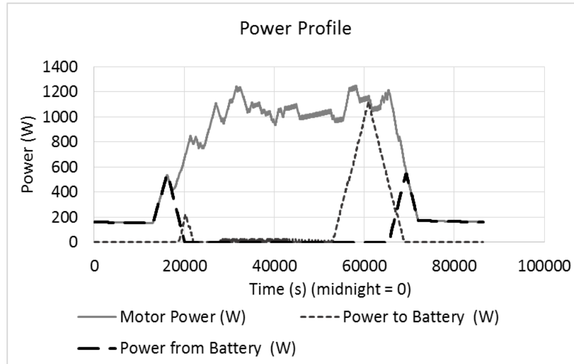

Fig. 3 Zoomed Power Profiles for Default Case

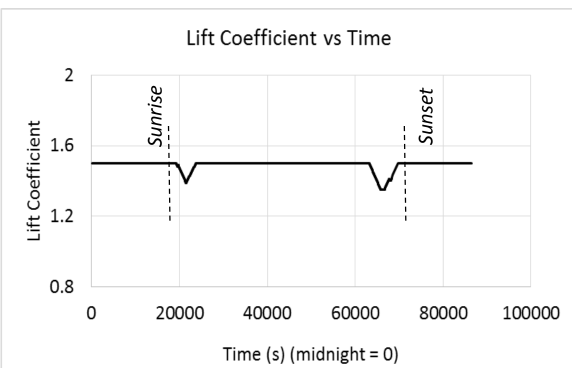

Fig. 4 Lift Coefficient vs. Time for Default Case

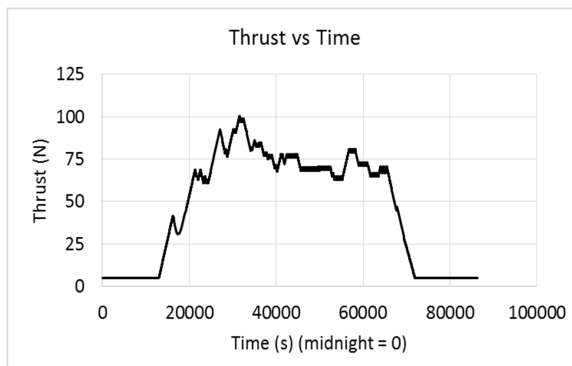

Fig. 5 Thrust vs. Time for Default Case

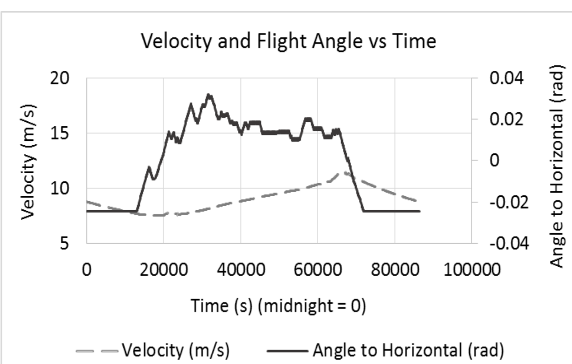

Fig. 6 Velocity and Angle for Default Case

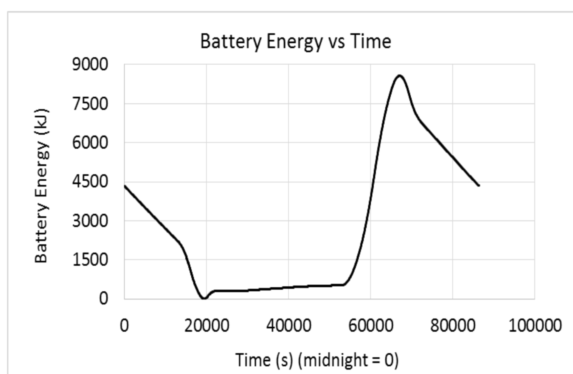

Fig. 7 Battery Energy vs. Time for Default Case

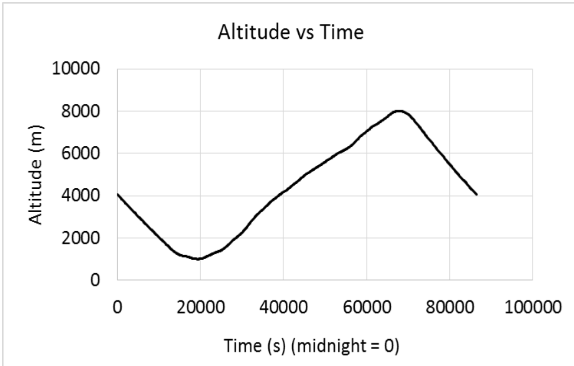

Fig. 8 Altitude vs. Time for Default Case

Figure 2 is interesting as it shows that the solar power available is much greater than that used for powering the plane. This is because the default case is day 180, close to the summer solstice in San Francisco. The power discharge from the battery is quite small at night, and is barely visible in Fig. 2. Therefore, a zoomed version of Fig. 2 without the solar power is shown in Fig. 3. This discharge is so small because the plane has a smooth descent (Fig. 8) while providing the minimum required thrust of 5 N. Periods of peak discharge occur before sunrise when the plane has hit the lower altitude limit and around sunset to preserve the altitude (Fig. 8). Figures 5 and 6 show that the flight angle is directly correlated to the thrust control variable. The angle is negative until sunrise and then increases as solar power is available. The velocity changes little throughout the flight, although it hits a low around sunrise and then increases with thrust and altitude.

Figure 7 shows the battery energy steadily decreasing from sunset to sunrise due to the power discharge to the propeller. Since the maximum battery energy is minimized, the battery reaches dead storage level (indicated by 0 kJ) slightly after sunrise, when there is enough solar power to provide sufficient thrust. The charging of the battery occurs mainly towards the end of the day, as seen in the battery energy chart in Fig. 7. However, I ran some trials by forcing an earlier start to the charging and realized that the minimum battery capacity did not change, indicating the presence of multiple solutions. The reason for this is simple. As there is so much excess solar power available on the default day, which is in the middle of summer, there are multiple charging patterns that result in the same objective function. If the model were run on a day with less sunlight, such

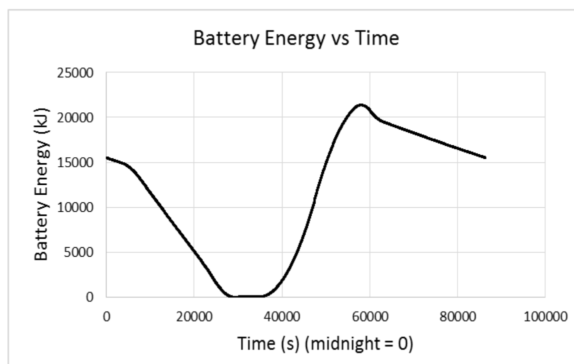

Fig. 9 Battery Energy on Winter Solstice (37° N)

as the winter solstice, the solver would have far less freedom and charging would begin earlier on in the day. Figure 9 shows the battery energy graph when the model is run on the winter solstice. Predictably, with less sunlight, the battery must start charging much sooner in the day to ensure that it has sufficient energy for the night. As constraints on the model are tightened, the issue of multiple solutions becomes far less problematic. However, the solution method performs reliably even if the objective function is flat over a range of values.

Figure 8 shows the altitude decreasing throughout the night to minimize energy use and then increasing throughout the day in order to gain potential energy from solar power. Figure 4 shows that the lift coefficient is relatively constant except for drops when the upper or lower bounds in altitude are reached. At these points, the solar flux is just barely enough to provide sufficient thrust. As the plane reaches the upper altitude limit, the lift coefficient is lowered to minimize lift induced drag. As night proceeds, the lift coefficient increases to reduce the fall rate, thus minimizing the battery energy requirement. Likewise, at the lower altitude limit, the solar flux is just enough to keep the altitude stable, so lowering the lift coefficient reduces the drag, thus minimizing battery use. Once sufficient solar flux is available, the lift coefficient increases to enable increase in altitude. When the model is optimized with all altitude restrictions removed, the two drops in the lift coefficient disappear.

Case 1 in Table 1 shows the effect of the altitude range on the objective function. As the airplane is given more altitude flexibility, the required energy storage steadily decreases. Since the plane is allowed to fly higher, it uses the solar energy available in the day to gain altitude. This allows for more room to descend in the night and thus less energy needs to be stored in the battery. For Case 2, the objective function values match what one would expect, as on the spring and fall equinox there is similar amount of sunlight. The summer solstice has maximum sunlight, so minimal energy needs to be stored in the battery. In contrast, for the winter solstice the greatest amount of energy needs to be stored. Figure 10 shows that a higher percentage of solar power is used on the winter solstice than on the default day (Fig. 2). Case 3

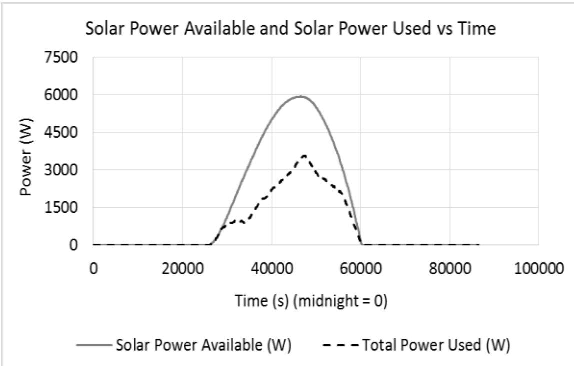

Fig. 10   Power Use at Lat 37° N - Winter Solstice

shows the effect of latitude. The battery energy storage needed is lower in San Francisco because the default case is the 180th day of the year; therefore, solar flux availability drops as we move south to the equator. For Case 4, the objective function is larger when the panel efficiency drops to 22 % (same as Solar Impulse [9]). Cases 5 and 6 show predictably that as payload increases or battery efficiency reduces, the needed energy storage increases. The battery efficiency study shows a reasonable safety margin, allowing for battery degradation from the default case.

Investigation of Limiting Cases: Results and Methods

I also established the zone of feasibility for continuous operation of the solar aircraft. Since the availability of solar power is the most important factor, I studied cases on the winter solstice in the northern hemisphere, and found that the maximum latitude at which the plane can fly continuously is

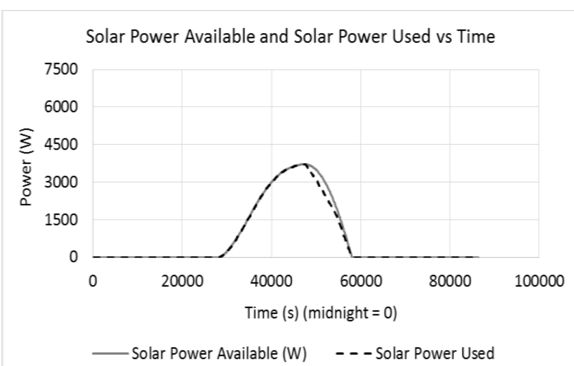

Fig. 11   Power Use at Lat 47.5° N - Winter Solstice

47.5° N, at which the required minimum battery capacity is 24918 kJ. All other parameters were kept fixed as per the default case shown in Table 1. Figure 11 compares the solar power available to the solar power used at latitude 47.5° N. Comparing this to the graph at latitude 37° N (Fig. 10), we can see that the graphs of the power available and the power used almost completely overlap in Fig. 11, as the plane is using nearly all of the solar power available for both propulsion and storing energy for use at night.

At latitudes higher than 47.5° N, solar power is insufficient to sustain continuous flight. Thus, on the winter solstice, this airplane would not be able to fly above Seattle, Washington. Regarding solar panel efficiency, even at an efficiency as low as 14% with all other parameters kept fixed as per the default case, the airplane could remain in the air continuously on the winter solstice at 37° N, thus enabling use of cheaper solar panels.

Computational Results

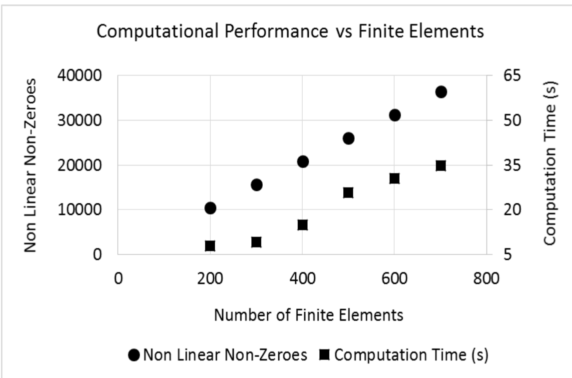

Fig. 12 Computation Time vs. Finite Elements

The simultaneous method I implemented proved robust and computationally efficient for the optimization. All the cases described above were solved with ease, especially due to enhancements made through testing on the glider problem. The computational performance of the solar aircraft model is reported in Fig. 12. As the number of finite elements increases, the degrees of freedom and nonlinear non-zeroes increase; however, even for 500 finite elements the computation time is only about 26 seconds. Low computation time is important for the applicability of my method to larger models and allows for quick readjustment of the trajectory should disturbances occur.

Conclusions and Future Work

The solution method I implemented for solving optimal control problems has proven to be robust and computationally efficient. The solar aircraft optimization runs, in which four control profiles are optimized, take less than 30 seconds for most cases, and by using the simultaneous integration and optimization method, the drawbacks of repeated integration of the differential equations are avoided.

The solar aircraft optimization model formulated is also the first comprehensive model that can be used in two ways. In the first step, the model can be used to determine the minimum battery capacity needed before launch. Following the launch, the model can perform as a control system to ensure that the flight stays on the optimal trajectory for continuous flight. I hope that this model will be used by future researchers to test their solution methods.

There is much more work that needs to be done. On the solver side, the effect of non-linearities on robustness needs to be further investigated. The solar airplane model must be enhanced by accounting for environmental factors such as clouds, rain, wind, and humidity. Regarding multiple solutions, the effect of additional constraints and limits on the control variable profiles must be evaluated. The current model is computationally efficient, but allows for limited motion about the launch point. This is not a major problem since the plane rarely reaches speeds above 50 *km/h*. However, the effect of the earth's rotation on the cycle time as well as Coriolis force effects and solar flux changes for more extensive north-south motion should be considered if the range of motion is expected to be wide. The effect of the tilt of the solar panel on the solar flux should also be considered in future work, as this could be a factor in take-off and landing. Furthermore, it is also important to consider better battery power output models in future work. I have recently joined a team of Stanford graduate students to design and build a small solar aircraft using the optimization method and plan to further improve the model and the solution algorithm in the process. I hope this project will help us evaluate the opportunities and practical limitations of continuous solar flight.

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