

A Generalized Herfindahl-Hirschman Index

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I. Personal Section

Economics was a subject I first discovered in high school—a class that all students were required to take which inexplicitly became my favorite class of the year. I was enthralled by the idea of using mathematics to model the behavior of individuals and firms across the country. Until I met my mentor Professor Cheng-Zhong Qin, that is how I saw economics: a collection of simple and elegant theories that described all of economic activity.

In truth, however, the complexities of the modern economy far outpace the mainstream theories economists use to describe it, most of which were developed before the mid-20th century. The great struggle of the next generation of economic researchers is not to develop new theories, but to adapt existing theory to the 21st century. Only this process can reconcile the original goal of economics—to be a model of the global economy—with the realities of our world.

My mentor's work focused upon an increasingly popular business organization technique known as “cross-holdings”; he had dedicated years of work theorizing on its effect on competition. However, I was interested not so much in the groundbreaking theories surrounding cross-holdings, but rather in its implications on existing competition theory. If emerging literature was correct, current models of competition were grossly inaccurate. Of particular concern to me was that the index used by regulatory bodies to evaluate concentration in a particular industry

dramatically underestimated competition. Perhaps in time brand-new models of competition would develop to correct these error, but it is unclear when, if ever, this would occur.

The purpose of my research was to be a bridge between these two halves: the bleeding edge of economic research and the models of today. I wanted to incorporate the findings of researchers like my mentor into existing theories, and thereby bring together economics and reality.

Nominally, my research occurred in an office across the hall from my mentor's UC Santa Barbara office. It would be more accurate to say that my research was born from the constant effort of more than two months. I thought about my research during sports practice; I made critical progress on a cross-country plane flight; I turned over ideas in the back of my head during dinner. As a result of the total dedication of my life, the final product feels more alive to me than any other economic paper: within it are countless hours of thinking, working, and revising, all captured within words and mathematics.

Although my project is not technically difficult and did not require the use of mathematics beyond simple single-variable calculus, it required me to apply mathematical concepts in new and creative ways. Frequently, the project stalled and resisted my attempts to make further progress. But through perseverance, every obstacle—no matter how frustrating—could be overcome. The hardest thing about finishing such a project, especially for someone who had never done anything similar in the past, is simply to keep working on it. If you believe in yourself, your project will be successful—no matter what the results are.

II. Research Section

Economics students everywhere are familiar with the famous Herfindahl-Hirschman Index (HHI), which is used to quantify the amount of competition in a particular industry.

$$H = \sum s_i^2$$

Where s_i represents the market share (the ratio of an individual firm's sales to the sales of the entire industry) of each firm. This simple yet powerful formula has been the preferred method of calculated market concentration of antitrust authorities—in fact, the US Federal Trade Commission relies heavily upon HHI analysis in antitrust cases—since its creation over 66 years ago. The index approaches 0 in situations of perfect competition and 1 in the case of perfect monopoly, in which one firm has 100% market concentration.

This index falls short when considering the modern technique of cross-holding. Cross-holdings exist when one business holds a non-voting stake in a direct competitor. Because these stakes are non-voting, they are allowed under current antitrust regulations; since sales are unaffected, they also fail to affect the HHI. However, recent economic literature has established that cross-holdings have a substantial impact on competition. In fact, in extreme scenarios it is possible for a multi-firm industry with cross-holdings to operate indistinguishably from a pure monopoly.

Our project has two parts: first, we use conditions of Cournot Equilibrium (a special case of oligopoly) to derive a generalized form of the HHI, which we call the Generalized Herfindahl-Hirschman Index (GHHI). We dedicate a substantial portion of the paper to showing a number of mathematical properties of the GHHI which make it suitable as a replacement for the HHI.

In the second portion of the paper, we apply the GHHI to the US financial services industry, an industry with a history of substantial cross-holdings, to evaluate the seriousness of the anti-competitive effects of cross-holdings in real-world markets.

1) Derivation of GHHI

We take the perspective of the manager of a firm j seeking to maximize profits in conditions of Cournot Equilibrium. The resulting equation is

$$\sum_i \gamma_{ij} \pi^i = \sum_i \gamma_{ij} \sum_k \beta_{ik} \pi_k = \sum_i \gamma_{ij} \sum_k \beta_{ik} [P(X)x_k - C(x_k)]$$

where x_j , s_j , and π_j are a given firm j 's output, market share, and profit, respectively. X is the total industry output, $P(x)$ is the inverse demand function, and $C(x)$ is the cost function. β_{ij} represents the share of the profits of firm j owes to firm i , and γ_{ij} represents the degree of control owner i exerts over the management of firm j . From Cournot equilibrium, we also know that

$$\frac{d\pi_j}{dx_j} = 0$$

Using these equations together with conditions of silent financial interests, we are able to simplify the above equation to

$$\sum_j s_j \frac{P - C'(x_j)}{P} = \frac{-1}{\tau} \left[\sum_{j,k} s_j s_k \frac{\beta_{jk}}{\beta_{jj}} \right]$$

where τ is the price-elasticity of demand. We define the GHHI to be the bracketed portion, i.e.

$$H_G = \sum_{j,k} s_j s_k \frac{\beta_{jk}}{\beta_{jj}}$$

There are a number of interesting and important attributes of the GHHI that must be noted.

First and foremost, in the absence of cross-holdings the GHHI is identical to the HHI. This makes it truly a generalization of the HHI to conditions where cross-holdings exist.

Another very important characteristic to note is that if we make the reasonable assumption that every firm retains plurality control over itself (that is, the managers of every firm retain the biggest share of profits), the GHHI has the same bounds as the original HHI.

These bounds match up nicely with the behavior of the original HHI. As before, a GHHI of 0 represents perfect competition. In addition, it is easy to verify that the GHHI approaches 1 in both pure monopoly and *de facto* monopoly achieved through cross-holdings; in other words, the GHHI successfully takes the anti-competitive effects of cross-holdings into account.

2) Application to Financial Services Industry

The second part of the project was an empirical analysis conducted on 17 of the 25 largest US asset management firms. These firms were used to analyze both the extent of the impact their extensive cross-holding relations with each other have on competition, as well as to gain insight on the market forces which drive firms to form cross-holdings with each other.

To summarize our results, the presence of cross-holdings in the financial services industry has a noticeable impact on competition. The traditional HHI is 0.0895, which would indicate a highly competitive industry, but with cross-holdings taken into account it rises to 0.1382—more than a 50% increase—indicating “mild concentration.”

While the immediate findings of this empirical analysis are relatively muted, the implications are profound. The substantial rise in concentration when cross-holdings are accounted for raises the troubling possibility that firms could potentially use cross-holdings as a way to undergo “sneaky mergers”—in other words, *de facto* mergers which escape the notice of antitrust authorities because they employ unregulated channels. In fact, emerging literature suggests that, intentionally or not, this may be the case in the airline and European auto industries.

Moving forward, we hope to conduct additional tests on the GHHI to verify its accuracy as a method of quantifying concentration in intermediate scenarios. This process of revising old economic methods to adapt to changing conditions promises to provide new and powerful tools for academia and future antitrust authorities.

References

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