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Jason Yang

UChicago *E = mc^2: A High School Science Journal* article entry

Personal Section:

I started coding on Scratch during middle school, initially making very rudimentary animations and games, then some 3D graphics projects. Around my sophomore year in high school, I started to do competitive programming on platforms like TopCoder and the USA Computing Olympiad, and focus on computer algorithms. Eventually, I was accepted into the MIT PRIMES Computer Science program, where I research under mentor Jun Wan. After exploring a few different topics, we settled on one inspired by a question I thought of in the field of competitive programming. There is this data structure called a “segment tree with lazy propagation”, which allows one to store a list of numbers, then “update” an arbitrary range in the list (ex. Add all numbers in the range by some constant value), and also “query” an arbitrary range in the list (ex. Find the minimum of all numbers in the range). The segment tree performs these two operations much faster than a brute force method would. When I first learned this data structure, my question was: is there a 2D analog of it?

While I didn’t have to learn additional mathematics to get the main results in my project, I did have to come up with clever proofs, first in the form of small steps to solving our main question, and then in coming up with a big step. I would say this project definitely made science and math more vivid for me because my project connected the topic I originally investigated to another problem in computer science: matrix multiplication. Both the specific variant of matrix multiplication relevant to our research, namely the “min-plus” variant, as well as standard

matrix multiplication happen to be major efficiency bottlenecks in many other problems involving communication networks, machine learning, and computer graphics. In particular, I have become especially interested in matrix multiplication as a result of this project, and am currently reading papers and running computer simulations about the topic in my free time.

For advice to high-schoolers interested in math and science, I think you should pursue competitions, research programs, and/or other opportunities to get a lot out of your interests. At the same time, I think it also really helps to be personally interested in some unsolved problems in your fields, which can help attach a high goal and a strong purpose for studying in those fields; for me, it's fast matrix multiplication in the field of computer science.

Research section:

The main problem of our project was investigating whether or not there was an efficient 2D analog to the segment tree. Here, instead of updating and querying arbitrary ranges of a list of numbers, we want to update and query arbitrary submatrices of a matrix of numbers. When updating a submatrix, we add all numbers in the submatrix with an arbitrarily chosen constant value; when querying a submatrix, we find the minimum of all numbers in the submatrix. We wanted to see whether there was a data structure that could efficiently perform each of these operations being repeatedly done one after the other, where each update can affect future queries. The answer we found to this question is a conditional no if a specific long-standing conjecture is assumed. The conjecture is the "All-Pairs Shortest Path Conjecture", which states that the "All-Pairs Shortest Path" problem cannot be solved in "truly subcubic" time. Since at least the 1970s, it has been known that the "All-Pairs Shortest Path" problem has equal difficulty

to computing a certain operation on matrices, called “min-plus matrix multiplication”, which is similar to the standard matrix multiplication in linear algebra but where the summation is replaced with the minimum() function and the multiplication is replaced with addition. Also since the 1970s, this conjecture has remained open, and not too much progress has been made towards refuting it. The key to achieving our research result was realizing that if there was an efficient 2D segment tree, then it could perform min-plus matrix multiplication in truly subcubic time, which would refute the conjecture. Therefore, if the conjecture is true, which many believe to be the case, then an efficient 2D segment tree is impossible.